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Capitolo W77
promtuario: meccanica quantistica

Contenuti delle sezioni

11 pagine

W77 0.01 Gran parte delle definizioni e dei risultati della meccanica quantistica sono esprimibili mediante formule che spesso risultano alquanto elaborate.
Fine di questo capitolo è una presentazione di definizioni e risultati della meccanica quantistica che possa costituire un conveniente riferimento compatto per lo studio e le applicazioni della disciplina.

Le formule che seguono si possono distinguere in due categorie.
Quelle della prima categoria riguardano sistemi quantici specifici e strutture generali per la disciplina.
Quelle della seconda classe, inserite per rendere il capitolo un repertorio più autonomo e completo, toccano varie nozioni matematiche (dell'analisi, della geometria e della fisica matematica) che possono essere viste come preliminari delle formule più strettamente quantistiche.

W77 a. valori di aspettazione in una dimensione

W77 a.01 Funzione d'onda $\Psi(x,t)$ grandezze $f(x,t)$ e $F(x,p,t)$

$$\big\langle f(x,t)\big\rangle \;=\; \int_{-\infty}^{\infty} \Psi^*(x,t)\, f(x,t)\cdot\Psi(x,t) \qquad \big\langle F(x,p,t)\big\rangle \;=\; \int_{-\infty}^{\infty} \Psi^*(x,t)\, F_{op}\left(x, -\mathrm{i}\hbar\frac{\partial}{\partial x}, t\right)\cdot\Psi(x,t)$$

$$\langle x\rangle \;=\; \int_{-\infty}^{\infty} \Psi^*(x,t)\, x\cdot\Psi(x,t) \qquad \langle p\rangle \;=\; -\mathrm{i}\hbar\int_{-\infty}^{\infty} \Psi^*(x,t)\,\frac{\partial}{\partial x}\Psi(x,t)$$

$$\langle E\rangle \;=\; \mathrm{i}\hbar\int_{-\infty}^{\infty} \Psi^*(x,t)\,\frac{\partial}{\partial t}\Psi(x,t) \qquad \Delta x \;=\; \sqrt{\langle x^2\rangle - \langle x\rangle^2} \qquad \Delta p \;=\; \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

potenziale pari $\implies \langle x\rangle \;=\; \langle p\rangle \;=\; 0\ ,\ \Delta x \;=\; \sqrt{\langle x^2\rangle}\ ,\ \Delta p \;=\; \sqrt{\langle p^2\rangle}$

W77 b. coordinate sferiche

W77 b.01

$$\begin{aligned} r &= \sqrt{x^2+y^2+z^2} & 0 \leq r < \infty & \qquad x = r\cos\phi\sin\theta \\ \theta &= \arccos z/r & 0 \leq \theta < \pi & \qquad y = r\sin\phi\sin\theta \\ \phi &= \arctan y/x & 0 \leq \phi < 2\pi & \qquad z = r\cos\theta \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial x} &= \cos\phi\sin\theta\,\frac{\partial}{\partial r} + \frac{1}{r}\cos\phi\cos\theta\,\frac{\partial}{\partial\theta} - \frac{1}{r}\,\frac{\sin\phi}{\sin\theta}\,\frac{\partial}{\partial\phi} \\ \frac{\partial}{\partial y} &= \sin\phi\sin\theta\,\frac{\partial}{\partial r} + \frac{1}{r}\cos\phi\cos\theta\,\frac{\partial}{\partial\theta} - \frac{1}{r}\,\frac{\sin\phi}{\sin\theta}\,\frac{\partial}{\partial\phi} \end{aligned}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial^2}{\partial\phi^2}$$

$$\mathsf{p}\cdot\mathsf{p} = -\hbar^2\nabla^2$$

W77 c. oscillatore armonico

W77 c.01 Potenziale $V(x) = \frac{1}{2} K x^2$ con $K > 0$

$$\nu_c = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad \alpha = \left(\frac{m K}{\hbar^2}\right)^{1/4} \qquad E_n = \left(n + \frac{1}{2}\right) \mathtt{h}\, \nu_c$$

$n = 0, 1, 2, \ldots \qquad \psi_n(x) = N_n \,\mathsf{Hrmt}_n(\alpha\, x)\, \mathsf{e}^{-\frac{1}{2}\alpha^2 x^2} \qquad \xi := \alpha\, x \qquad N_n = \sqrt{\frac{\alpha}{\pi^{1/2}\, 2^n\, n!}}$

$\mathsf{Hrmt}_n(\xi) = (-)^n \mathsf{e}^{\xi^2} \frac{\mathrm{d}^n}{\mathrm{d}\xi^n}\, \mathsf{e}^{-\xi^2}$ polinomio di Hermite di grado n

$\mathsf{Hrmt}_0(\xi) = 1$, $\mathsf{Hrmt}_1(\xi) = 2\,\xi$, $\mathsf{Hrmt}_2(\xi) = 4\,\xi^2 - 2$, $\mathsf{Hrmt}_{n+1}(\xi) = 2\,\xi\, \mathsf{Hrmt}_n(\xi) - 2\,n\, \mathsf{Hrmt}_{n-1}(\xi)$

$\psi_0 = N_0\, \mathsf{e}^{-\xi^2/2}$, $\psi_1 = N_1\, \xi\, \mathsf{e}^{-\xi^2/2}$, $\psi_2 = N_2 \left(1 - 2\,\xi^2\right) \mathsf{e}^{-\xi^2/2}$,

$\psi_3 = N_3 \left(3\,\xi - 2\,\xi^3\right) \mathsf{e}^{-\xi^2/2}$, $\psi_4 = N_4 \left(3 - 12\,\xi^2 + 4\,\xi^4\right) \mathsf{e}^{-\xi^2/2}$

W77 d. valori di aspettazione in tre dimensioni

W77 d.01 Funzione d'onda $\psi(r,\theta,\phi)$ grandezza $f(r,\theta,\phi)$

$$\big\langle f(r,\theta,\phi)\big\rangle \;=\; \int_0^{+\infty} \mathrm{d}r\ r^2 \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi\ \psi^*(r,\theta,\phi)\, f(r,\theta,\phi)\, \psi(r,\theta,\phi)$$

Simmetria sferica, cioè $\psi \;=\; \psi(r)$

$$\langle x\rangle \;=\; \langle y\rangle \;=\; \langle z\rangle \;=\big\langle p_x\big\rangle \;=\big\langle p_y\big\rangle \;=\big\langle p_z\big\rangle \;=\; 0$$

$$\big\langle x^2\big\rangle \;=\big\langle y^2\big\rangle \;=\big\langle z^2\big\rangle \;=\; \frac{1}{3}\big\langle r^2\big\rangle$$

$$\big\langle {p_x}^2\big\rangle \;=\big\langle {p_y}^2\big\rangle \;=\big\langle {p_z}^2\big\rangle \;=\; \frac{1}{3}\langle p^2\rangle \;=\; -\frac{\hbar^2}{3}\big\langle \nabla^2\big\rangle$$

$$\big\langle f(r)\big\rangle \;=\; \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} \mathrm{d}r\ r^2\, \psi^*\, f(r)\, \psi(r) \;=\; 4\,\pi \int_0^{+\infty} \mathrm{d}r\ r^2 f(r)\, |\psi(r)|^2$$

$$\frac{1}{3}\big\langle p^2\big\rangle \;=\; -\frac{\hbar^2}{3} \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} \mathrm{d}r\ r^2\, \psi^*\, \frac{1}{r^2}\, \frac{\partial}{\partial\, r}\left[r^2\, \frac{\partial}{\partial\, r}\psi(r)\right]$$

$$\;=\; 4\,\pi \int_0^{+\infty} \mathrm{d}r\ r^2\, \psi^*\, \frac{1}{r^2}\, \frac{\partial}{\partial\, r}\left[r^2 \frac{\partial}{\partial\, r}\psi(r)\right]$$

W77 e. atomo idrogenoide

W77 e.01 $\psi_{n,l,m}(r,\theta,\phi) \;=\; R_{n,l}\, Ysph_{l,m}(\theta,\phi) \;=$

$$= -\sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}}\; \mathsf{e}^{-\rho/2}\rho^l\, \mathsf{Plgdra}_{n+l}^{2l+1}(\rho)\; Ysph_{l,m}(\theta,\phi)$$

$n \;=\; 1,2,3,...\quad l \;=\; 0,1,2,...,n-1 \quad m \;=\; -l,-l+1,...,0,...,l-1,l$

$a_0 \;=\; \dfrac{\hbar^2}{\mathsf{e}\,M_e} \quad \rho \;=\; \dfrac{2Zr}{na_0} \quad , \quad E_n \;=\; -\dfrac{Z^2\mathsf{e}^4\, m_e}{2\hbar^2 n^2} \;=\; -\dfrac{Z^2\,\mathtt{h}\,\mathtt{e}}{n^2} \quad , \quad R \;=\; \dfrac{m_e\,\mathsf{e}^4}{4\,\pi\hbar^3 c} \;=\; \dfrac{2\,\pi^2\, m_e\,\mathsf{e}^4}{\mathtt{h}^3\,\mathtt{c}}$

$Llag_\beta^\alpha(\rho) \;=\; \dfrac{\mathrm{d}^\alpha}{\mathrm{d}\rho^\alpha}\left[\mathsf{e}^\rho \dfrac{\mathrm{d}^\beta}{\mathrm{d}\rho^\beta}\left(\rho^\beta\,\mathsf{e}^{-\rho}\right)\right]$ polinomio di Laguerre

$\beta \;=\; 0,1,2,... \quad \alpha \;=\; 0,1,2,...,n$

$Ysph_{l,m}(\theta,\phi) \;=\; (-)^{\frac{|m|-m}{2}} \sqrt{\dfrac{2l+1}{4\pi}\dfrac{(l-|m|)!}{(l+|m|)!}}\, \mathsf{Plgdra}_l^{|m|}(\cos\theta)\,\mathsf{e}^{im\phi}$ funzione sferica di superficie

$\mathsf{Plgdra}_l^{|m|}(\xi) \;=\; (-)^m\,(1-\xi^2)^{\frac{|m|}{2}}\, \dfrac{\mathrm{d}^{|m|}\mathsf{Plgdr}_l(\xi)}{\mathrm{d}\xi^{|m|}}$ funzioni associate di Legendre

$\mathsf{Plgdr}_l(\xi) \;=\; (-)^l\,\dfrac{1}{2^l l!}\,\dfrac{\mathrm{d}^l}{\mathrm{d}\xi^l}(1-\xi^2)^l$ polinomio di Legendre di grado l

Strato K stato $1s$ $\psi_{100} \;=\; \dfrac{1}{\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \mathsf{e}^{-\rho/2}$

Strato L stati:

$2s \quad \psi_{200} \;=\; \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}(2-\rho)\,\mathsf{e}^{-\rho/2}$

$2p \quad \psi_{210} \;=\; \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}\rho\,\mathsf{e}^{-\rho/2}\cos\theta$

$2p \quad \psi_{21\pm1} \;=\; \mp\dfrac{1}{8\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}\rho\,\mathsf{e}^{-\rho/2}\sin\theta\,\mathsf{e}^{\pm \mathsf{i}\phi}$

W77 f. momenti angolari ed effetti magnetici

W77 f.01 momento angolare orbitale $\vec{L} := \vec{r} \times \vec{p} \quad L_z = x\,p_y - y\,p_x, \dots$

$$L^2{}_{(op)} = -\hbar^2 \nabla^2{}_a = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right] \qquad L_{z\,(op)} = -\mathrm{i}\hbar \frac{\partial}{\partial\phi}$$

autofunzioni degli atomi idrogenoidi senza spin $\quad \psi_{nlm}(r,\theta,\phi)$

$L^2\,\psi_{nlm} = l(l+1)\,\hbar^2\,\psi_{nlm} \qquad l = 0, 1, \dots, n-1$

$L_z\,\psi_{nlm} = m\,\hbar\,\psi_{nlm} \qquad m = -l, -l+1, \dots, 0, \dots, l-1, l$

orbitali corrispondenti: $l = 0 \Rightarrow s \quad l = 1 \Rightarrow p \quad l = 2 \Rightarrow d \quad l = 3 \Rightarrow f$

$l = 0 \iff \psi_{n00}(r,\theta,\phi) = \psi_{n00}(r) \iff$ simmetria sferica

$$\Longrightarrow \langle x \rangle = \langle y \rangle = \langle z \rangle = \langle \vec{r} \rangle = \langle p_x \rangle = \langle p_y \rangle = \langle p_z \rangle = \langle \vec{p} \rangle = 0$$

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = 1/3 \langle r^2 \rangle \qquad \langle p_x{}^2 \rangle = \langle p_y{}^2 \rangle = \langle p_z{}^2 \rangle = 1/3 \langle p^2 \rangle = -\frac{\hbar^2}{3} \langle \nabla^2 \rangle$$

$$\langle f(r) \rangle = 4\pi \int_0^{+\infty} \mathrm{d}r\; r^2 \psi^*(r) f(r) \psi(r) \quad , \quad \langle \nabla^2 \rangle = 4\pi \int_0^{+\infty} \mathrm{d}r\; \psi^* \frac{\partial}{\partial r} \left[r^2 \frac{\partial}{\partial r} \psi(r) \right]$$

momento di dipolo magnetico orbitale $\quad \vec{\mu}_l = -\dfrac{g_l\,\mu_b}{\hbar} \vec{L}$

$\mu_b = \dfrac{e\hbar}{2m_e} = 0.927 \cdot 10^{-23} \mathrm{Am}^2$ magnetone di Bohr; $\qquad g_l = 1$ fattore orbitale

energia di interazione dipolo - campo magnetico $\quad -\vec{\mu}_l \cdot \vec{B}$

momento angolare intrinseco o spin $\vec{S}$

autofunzioni di L^2, L_z, S^2, S_z $\quad : \psi_{nlmsm_s}$

$S^2\,\psi = s(s+1)\,\hbar^2\,\psi \qquad$ per l'elettrone $s = \dfrac{1}{2}$

$S_z\,\psi = m_s\,\hbar\,\psi \qquad$ per l'elettrone $m_s = \pm\dfrac{1}{2}$

momento di dipolo magnetico di spin - campo magnetico $\quad \vec{\mu}_s = -\dfrac{g_s\,\mu_b}{\hbar} \vec{S}$, $g_s = 2$

energia di interazione spin - campo magnetico $\quad \pm g_s\,\mu_b\,\dfrac{B}{2}$

deviazione Δz di Stern-Gerlach di atomo di massa M e velocità v_x per il quale

$\dfrac{1}{2} M\,v_x{}^2 = 2\,K_B\,T$ che attraversa magnete di lunghezza X e gradiente $\dfrac{\partial B_z}{\partial z}$:

$$\Delta z = \pm \frac{\partial B_z}{\partial z} \frac{\mu_b\,X^2}{8\,K_B\,T}$$

momento angolare totale $\quad \vec{J} = \vec{L} + \vec{S} \quad$ autofunzioni ψ_{nlsjm_j}

$J^2\,\psi = j(j+1)\,\hbar^2\,\psi \qquad$ per l'elettrone $j = l - \dfrac{1}{2}, l + \dfrac{1}{2}$ (valori semidispari)

$J_z\,\psi = m_j \hbar\,\psi \qquad m_j = -j, -j+1, \dots, j-1, j$

$(\vec{S} \cdot \vec{L})\,\psi = \dfrac{\hbar^2}{2} \big[j(j+1) - l(l+1) - s(s+1) \big]\,\psi$

energia di interazione spin - orbita $\quad l \ge 1 \quad E_{SL} = \dfrac{1}{2\,m_e{}^2\,\mathsf{c}^2} \dfrac{1}{r} \dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r} \left(\vec{S} \cdot \vec{L} \right)$

per atomi idrogenoidi: $s = \dfrac{1}{2}$, $V = -\dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0} \dfrac{1}{r}$, $E_{SL} = \dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,m_e{}^2\,\mathsf{c}^2} \dfrac{1}{r^3} \left(\vec{S} \cdot \vec{L} \right)$

$$\langle E_{SL} \rangle_{j,l,s} = \frac{\hbar^2}{4\,m_e{}^2\,\mathsf{c}^2} \big[j(j+1) - l(l+1) - s(s+1) \big] \left\langle \frac{1}{r} \frac{\mathrm{d}\,V(r)}{\mathrm{d}r} \right\rangle$$

$$\langle E_{SL} \rangle_{j,l} = \frac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,m_e{}^2\,\mathsf{c}^2} \big[j(j+1) - l(l+1) - 3/4 \big] \left\langle \frac{1}{r^3} \right\rangle$$

$$= \frac{|E_n|\,\alpha^2\,Z^4}{n\,l\,(l+\frac{1}{2})\,(l+1)}\,\frac{1}{2}\,[j(j+1)-l(l+1)-3/4] \quad \left(|E_n| \;=\; \frac{13.607\,\mathrm{eV}}{n^2}\;,\;\alpha\;=\;\frac{\mathsf{e}^2}{4\epsilon_0\hbar c}\simeq\frac{1}{137}\right)$$

W77 g. integrali notevoli per la quantistica

W77 g.01 $\int_0^{+\infty} \mathrm{d}x \;\; x^n \, \mathsf{e}^{-x} \;\; = \;\; n!$

$$I_m \;\; := \;\; \int_{-\infty}^{\infty} \mathrm{d}x \;\; x^m \, \mathsf{e}^{-x^2} \;\; = \;\; \frac{m-1}{2} \, I_{m-2}$$

$$I_0 \;=\; \sqrt{\pi} \quad , \quad I_1 \;=\; \sqrt{\pi} \quad , \quad I_2 \;=\; \sqrt{\frac{\pi}{2}} \quad , \quad I_4 \;=\; \sqrt{\frac{\pi}{2}} \quad , \quad I_{2\,n} \;=\; \frac{(2n-1)!!}{2^n} \sqrt{\pi}$$

$$\int_0^{\pi} \mathrm{d}x \;\; \cos^2 x \;\; = \;\; \int_0^{\pi} \mathrm{d}x \;\; \sin^2 x \;\; = \;\; \frac{\pi}{2} \quad , \quad \int_0^{\pi} \mathrm{d}x \;\; \cos^2 x \; \sin^2 x \;\; = \;\; \frac{\pi}{8}$$

$$\int_0^{\pi/2} \mathrm{d}x \; \sin^n x \; \cos x \;\; = \;\; \int_0^{\pi/2} \mathrm{d}x \; \sin x \; \cos^n x \;\; = \;\; \frac{1}{n+1}$$

$$\int_0^{\pi/2} \mathrm{d}x \; \sin^n x \; \cos^k x \;\; = \;\; \frac{n!\,k!}{2(n+k+1)!}$$

W77 h. numeri quantici e codifiche convenzionali

W77 h.01 $n = 1, 2, 3, ...$ $\quad l = 0, 1, ..., n-1$
orbitali corrispondenti: $l = 0 \Rightarrow s$, $l = 1 \Rightarrow p$, $l = 2 \Rightarrow d$, $l = 3 \Rightarrow f$

$m = -l, -l{+}1, ..., 0, ..., l{-}1, l$ $\quad m_s = -s, ..., s$ $\quad j = l{-}s, ..., l{+}s$ $\quad m_j = -j, -j{+}1, ..., j{-}1, j$

per atomi idrogenoidi, alcalini e simili $s = \dfrac{1}{2}$, $m_s = \pm\dfrac{1}{2}$, $j = l - \dfrac{1}{2}, l + \dfrac{1}{2}$

W77 i. effetti magnetici

W77 i.01 energia di interazione spin - campo magnetico $\pm g_s\,\mu_b\,\dfrac{B}{2}$

ove $g_s = 2$, $\mu_b = \dfrac{e\,\hbar}{2\,m_e} \;=\; 0.927\cdot 10^{-23}\mathrm{J/T}(\;=\;\mathrm{A\,m^2})$ magnetone di Bohr

$\Longrightarrow$ B efficace che porta a spaz. spin-orbita $2\,\mu_b\,B \;=\; \big\langle\Delta E_{SL}\big\rangle \;=\; \mathtt{h\,c}\left(\dfrac{1}{l} - \dfrac{1}{l'}\right)$

deviazione Δz di Stern-Gerlach di atomo di massa M e vel. v_x $\left(\dfrac{1}{2}M{v_x}^2 \;=\; 2\,K_B\,T\right)$

che attraversa magnete di lunghezza X e gradiente $\dfrac{\partial B_z}{\partial z}$: $\Delta z \;=\; \pm\dfrac{\frac{\partial B_z}{\partial z}\,\mu_b\,X^2}{8\,K_B\,T}$

energia di interazione spin-orbita $l \geq 1$ $E_{SL} \;=\; \dfrac{1}{2\,{m_e}^2\,\mathtt{c}^2}\,\dfrac{1}{r}\,\dfrac{\mathrm{d}\,V(r)}{dr}\left(\vec{S}\cdot\vec{L}\right)$

per atomi idrogenoidi $s \;=\; \dfrac{1}{2}$, $V \;=\; -\dfrac{\mathtt{e}^2\,Z}{4\,\pi\,\epsilon_0}\left\langle\dfrac{1}{r}\right\rangle$, $E_{SL} \;=\; \dfrac{\mathtt{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathtt{c}^2}\,\dfrac{1}{r^3}\left(\vec{S}\cdot\vec{L}\right)$

$\langle E_{SL}\rangle_{j,l,s} \;=\; \dfrac{\hbar^2}{4\,{m_e}^2\,\mathtt{c}^2}\big[j(j+1) - l(l+1) - s(s+1)\big]\left\langle\dfrac{1}{r}\,\dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r}\right\rangle$

per atomi idrogenoidi $\langle E_{SL}\rangle_{j,l} \;=\; \dfrac{\mathtt{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathtt{c}^2}\,[j(j+1) - l(l+1) - 3/4]\left\langle\dfrac{1}{r^3}\right\rangle \;=$

$$\dfrac{|E_n|\alpha^2 Z^4}{nl(l+\frac{1}{2})(l+1)}\,\dfrac{1}{2}\big[j(j+1) - l(l+1) - 3/4\big]\quad\left(|E_n| \;=\; \dfrac{13.607\,\mathrm{eV}}{n^2}\ ,\ \alpha \;=\; \dfrac{\mathtt{e}^2}{4\,\epsilon_0\,\hbar\,\mathtt{c}} \simeq \dfrac{1}{137}\right)$$

spaziatura dei livelli dovuta a interazione spin-orbita $\langle\Delta E_{SL}\rangle \;=\; \langle\Delta E_{SL}\rangle_{j_1} - \langle\Delta E_{SL}\rangle_{j_2}$

risoluzione necessaria per osservare separazione SL

$$\dfrac{\delta\lambda}{\lambda} \;=\; \dfrac{\langle\Delta E_{SL}\rangle}{E_{\to n}}\quad\left(E_{\to n} \;=\; 13.607\,\mathrm{eV}\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)\right)$$