Title: UPS reliability analysis with non-exponential duration distribution

Author: A. Pievatolo, CNR-IMATI, Milano, Italy (*)
Via Ampere 56, 20131 Milano
Phone +39/0270643213, Fax +39/0270643212
marco@mi.imati.cnr.it

I. Valadè, Dipartimento di Elettrotecnica Politecnico di Milano, Italy
Piazza Leonardo da Vinci 32, 20133 Milano
Phone: +39/0223993752, fax: +39/0223993703
ivan.valade@polimi.it

Abstract: In this paper, the reliability performance of uninterruptible power systems (UPS) is studied. After the description of the protection system against anomalous conditions, a brief failure mode analysis is performed in order to define the fault tree referring to compensator output voltage. An analytical model able to deal with non-exponential life and repair time distributions is developed, using semi-Markov processes and assuming stochastic independence for the components of the system under study. The mean time between failure (MTBF) and the mean time to restoration (MTTR) of the compensator output voltage are then calculated exactly. Finally a mechanical bypass switch, which connects the load to the mains directly during long failures, is taken into account in the MTBF calculation via a simple approximation. Monte Carlo simulations are performed to validate the results of the analytical model and of the approximation.

Keywords: Semi-Markov process, life and repair time duration distribution, Monte Carlo method.

Main text

I. INTRODUCTION

Interruptions of the electrical power supply, even for a very short time, are correlated with expensive problems such as interruption in a manufacturing cycle, alteration of data in information technology systems and malfunctioning in electrical systems apparatuses. The quality of the power supply can be improved by using active compensation devices, which can be well-tried solutions such as the double-conversion UPS or innovative ones, such as the promising Unified Power Quality Conditioner (UPQC) [1]. In this work UPS is taken into consideration. The application of such devices makes an evaluation from a reliability point of view a matter of some importance. In order to carry out the reliability analysis, we study the failure mode of the components and the UPS protection system intervention directed to limit the failure consequences. The probabilistic analysis is then performed in steady state conditions, allowing us to evaluate the MTBF and MTTR of load supply. The reliability information has been obtained from the UPS’s in operation:

(*) Corresponding author
they include the frequencies of the failure modes and their mean repair times.

II. UPS DEVICE AND PROTECTION SYSTEM

The traditional double-conversion UPS consists of a rectifier, an inverter, an energy storage unit and a static transfer switch. In fig. 1, a UPS is shown with reference to the Italian distribution system.

The VSI (Voltage Source Inverter) rectifier considered is able to draw an input current with a unit power factor and low harmonic distortion, so that low losses and voltage harmonic distortion are produced in the mains. This rectifier, labelled “VSI” in figure 1, consists of the 3 phase IGBT (Insulated Gate Bipolar Transistor) bridge converter and of the filter needed to filter out the voltage and current harmonics at the converter switching frequency.

![Figure 1](image-url)

**Figure 1** Double conversion UPS with protection devices. Voltages $V_m$, $V_i$ and $V_{OUT}$ pertaining to phase “a” of the converter, are also shown.

The static transfer switch (STS) consists of the two three-phase static switches $SW_m$ and $SW_i$, each constituted in turn by two anti-parallel thyristors per phase, and allows the fast transfer of power from the inverter to the mains and vice versa.

The protection system of this device consists of the following [2]:
- overcurrent protection device (circuit breaker or fuse) at the A. C. (Alternating Current) input to the rectifier ($i_r$);
- control and limitation system of the current absorbed by the rectifier;
- circuit breaker or fuse for battery protection ($f_b$);
- overcurrent protection device placed at the inverter D.C. (Direct Current) input (fuses $f_{dc}$ and $f_b$);
- inverter output current limitation;
- fuses $f_{SW_m}$ and $f_{SW_i}$ protecting the thyristors of the STS. Fuses on the standby line must handle a short circuit current of 10 times the nominal load current during 1-5 cycles to obtain coordination with the load protection, while those following the inverter must be dimensioned only for the weak short circuit (S.C.) currents furnished by the inverter;
- desaturation circuits to protect the IGBTs from short circuit currents.

The following faults have been considered:
- in the semiconductor devices: S.C. or O.C. (Open Circuit), due to faults in the power or firing circuit;
- in the control logics of the components;
- short circuits in either the A.C. or the D.C. section;
- anomalous behaviour of the energy storage unit [3].

Table 1 shows the operational states determined for the components, considering the storage system autonomy unlimited.

<table>
<thead>
<tr>
<th>Section</th>
<th>Operational modes</th>
<th>Typical cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter</td>
<td>$I_{n1}$: correct operation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_{n2}$: short circuit</td>
<td>• power or firing circuit failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• output filter short circuit</td>
</tr>
<tr>
<td></td>
<td>$I_{n3}$: open circuit</td>
<td>• power or firing circuit failure</td>
</tr>
<tr>
<td></td>
<td>$I_{n4}$: control failure</td>
<td>• control logic failure</td>
</tr>
<tr>
<td>Rectifier</td>
<td>$R_{i1}$: correct operation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_{i2}$: short circuit</td>
<td>• power or firing circuit failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• input filter short circuit</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• DC capacitor short circuit</td>
</tr>
<tr>
<td></td>
<td>$R_{i3}$: open circuit</td>
<td>• power or firing circuit failure</td>
</tr>
<tr>
<td></td>
<td>$R_{i4}$: control failure</td>
<td>• control logic failure</td>
</tr>
<tr>
<td>Energy storage</td>
<td>$E_{s1}$: correct operation</td>
<td></td>
</tr>
<tr>
<td>unit</td>
<td>$E_{s2}$: short circuit</td>
<td>• cell short circuit</td>
</tr>
<tr>
<td>Static</td>
<td>$E_{s3}$: high impedance</td>
<td>• positive grid corrosion</td>
</tr>
<tr>
<td>transfer switch</td>
<td></td>
<td>• dry-out</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• plate sulphation</td>
</tr>
<tr>
<td></td>
<td>$SW_{s1}$: correct operation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$SW_{s2}$: $SW_{m}$ thyristor short circuit</td>
<td>• thyristor power circuit failure</td>
</tr>
<tr>
<td></td>
<td>$SW_{s3}$: $SW_{i}$ thyristor short circuit</td>
<td>• thyristor firing circuit failure</td>
</tr>
<tr>
<td></td>
<td>$SW_{s4}$: $SW_{m}$ thyristor open</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$SW_{s5}$: $SW_{i}$ thyristor open</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$SW_{s6}$: control failure</td>
<td>• control logic failure</td>
</tr>
</tbody>
</table>

Table 1 Operational modes of UPS components.

The analysis of failure mode effects of components has been carried out separately for the STS and the compensator, which has allowed us to construct the failure trees shown in figure 2.

The top event of the STS failure tree (figure 2.a) is the out-of-limits condition of the load voltage: in fact if this voltage goes below 90% of the nominal amplitude or above 110%, the sensitive load doesn’t work correctly and can be damaged. A short-circuit in a thyristor of $SW_{m}$, together with an out-of-limits condition of the mains voltage causes the top event because the weak short circuit current from the inverter is not capable of blowing the fuse $f_{SWm}$ in time. The case of a S.C. in a thyristor of $SW_{i}$ is different. In this case, a possible short circuit in the compensator output inverter is accompanied by a heavy short circuit current fed from the mains and the consequent blowing of fuse $f_{SWi}$. Therefore, this event doesn’t cause the top event and doesn’t appear in the tree.

The UPS device with the VSI rectifier should be capable of drawing from the mains the active power required by the load even when the residual voltage is very low increasing the absorbed current. It has been assumed that the current rating of the rectifier makes it possible to draw the full load power as long as the mains voltage exceeds 70% of the nominal. Below this point, the storage system must come into play.
In the following stochastic model, the mains is included as an additional component, along with the UPS components, which gives a representation of the power supply as seen from the UPS.

III. STOCHASTIC MODEL OF THE SYSTEM

The system is said to be in a failure state when compensator output voltage $V_{OUT}$ is out-of-limits; otherwise it is in a functioning state. This two state process is the result of a complicated combination of occurrences and can be adequately described by first modeling separately each component, and then using the fact that the components change states independently of each other. This is true because the following is usually valid:

-the main cascading failures, due principally to the circulation of short circuit currents in components other than the damaged one, are avoided due to the presence of protection devices which quickly isolate the damaged component. Overcurrent protection for the components of the active compensation devices is always incorporated into the design criteria of the UPS manufacturers in order to provide a safe and reliable installation;

\[
V_{OUT} \neq (0.9 - 1.1)V_n
\]

\[
V_i \neq (0.9 - 1.1)V
\]
-common cause failures are very limited because the device are usually made to work under suitable environmental conditions, as suggested in the user manuals. Besides the diffusion of advanced systems of telediagnostics and telecontrol of the UPS’s by the manufacturers makes it possible to carry out an adequate maintenance, both preventive and corrective. In fact, user-made maintenance errors are often reported to be the root cause for common cause failures. Finally, some protection devices are usually installed against the external disturbances that could cause common cause failures (particularly surge arresters);

-the repair of a component starts as soon as the fault has occurred, thanks to diagnostic signalling. The repair time includes the technician’s travelling time, fault identification and repair and putting the component back into service.

In order to evaluate the reliability parameters of interest, it is sufficient to study the components under steady state conditions.

A. Notation

The following notation has been used for the components:

\( C \) number of components in the system

\( X_c(t) \) state of component \( c \) at time \( t \)

\( x_c, i, j \) different ways of indicating a state taken by component \( c \)

\( K_c \) number of states that component \( c \) can take

\( T_{c,m} \) the time of occurrence of the \( m \)-th transition of component \( c \)

\( N_c(t) \) the number of transitions of component \( c \) up to time \( t \):

\[
N_c(t) = \sup\{n : T_{c,n} \leq t\}
\]

\( J_{c,m} \) the embedded Markov chain for component \( c \)

\( N_c(i) \) the process counting only the visits to state \( i \) of component \( c \)

\( P_{c}(i,j) \) probabilities of the transition matrix \( P_c \) of the embedded Markov chain for component \( c \)

\( D_{c,ij} \) time taken by component \( c \) to reach state \( j \) starting from \( i \) when it is decided that the next transition will be to \( j \)

\( F_{c,ij}(t) \) cumulative distribution function of \( D_{c,ij} \)

\( D_{c,x} \) amount of time spent by component \( c \) in state \( x_c \) from the transition to it to the next transition

\( S(t) \) state of the system at time \( t \)

\( s=(x_{j_1},...,x_c) \) a fixed state of the system

\( K \) number of states of the system

\( T_n \) the time of occurrence of the \( n \)-th transition of the system

\( N(t) \) the number of transitions of the system up to time \( t \):

\[
N(t) = \sup\{n : T_n \leq t\}
\]

\( D_s \) amount of time spent by the system in state \( s \) from the transition to it to the next transition

B. Semi-Markov model of the generic component \( c \)

At the moment \( T_{c,m} \) of the \( m \)-th transition of the component, the probability of any particular future behaviour of the process depends only on the current state [4] and thus the evolution of each component can be modelled as a semi-Markov process (see [5], chapt. 5 for the general theory of these processes).

This type of process is completely determined by the matrix of transition probabilities \( [P_c]_{i,j} \equiv P_c \) of the
embedded Markov chain, and by the conditional duration distributions \( F_{c,i,j}(t) \), which are taken absolutely continuous. The distribution of \( D_{c,i} \) is the mixture distribution

\[
F_{c,i}(t) = \sum_{j=1}^{K_c} P(i,j) \cdot F_{c,i,j}(t).
\]

(1)

For the components taken into consideration, \( P_c \) is an irreducible and positive recurrent transition matrix, and it admits an invariant probability distribution \( \{ \pi_{c,1}, \ldots, \pi_{c,K_c} \} \). Irreducibility and recurrence, plus absolute continuity of the duration distributions (so that they are non-lattice), give us a positive recurrent semi-Markov process with a proper steady state probability ([5], theorem 5.3, 5.8, and 5.14); by theorem 5.16 [5], considering also the existence of the invariant distribution for \( P_c \), the steady state probability of being in state \( i \) can be calculated as

\[
\Pr_{c}(i) = \lim_{t \to \infty} \Pr[X_c(t) = j | X_c(0) = j] \cdot \pi_{c,j} \cdot \mu_{c,j}, \quad \forall j
\]

(2)

where \( \mu_{c,j} = E(D_{c,j}) \). From now on, when taking limits as \( t \to \infty \), we omit the indication of the initial state.

We will also need the frequency of visits to state \( i \):

\[
F_{c,i}(t) = \lim_{\Delta \to 0} \frac{E(N_{c,i}(t+\Delta) - N_{c,i}(t))}{\Delta} = \lim_{\Delta \to 0} \frac{\Pr(X_c(t) = i \cap X_c(t+\Delta) \neq i)}{\Delta} \cdot \frac{\Pr(i)}{\mu_{c,i}}
\]

(3)

Being recurrent, the semi-Markov process \( X_c(t) \) is a regenerative one (i.e. it starts afresh each time it enters a fixed state, such as state \( i \)) and so it makes sense to define the mean recurrence time of state \( i \), \( \mu_{c,ii} \), that is, the mean time between two consecutive visits to state \( i \). Theorem 5.8 of [5] states that \( \Pr_{c}(i) = \mu_{c,i} / \mu_{c,ii} \) and substituting this into (3), we find the expression for frequency:

\[
F_{c,i}(t) = \frac{1}{\mu_{c,ii}}
\]

(4)

and we can find the mean recurrence time of any state from the frequency.

A particular case of a semi-Markov process is the continuous time Markov chain, that can be described via the transition rates \( \lambda_{c,i,j} \) [6]:

\[
\lambda_{c,i,j} = \lim_{\Delta \to 0} \frac{\Pr[X_c(t+\Delta) = j | X_c(t) = i]}{\Delta}
\]

(5)

which are independent of \( t \) by the Markovian property. It can be shown that:

\[
\mu_{c,j} = \frac{1}{\lambda_{c,j}}, \quad \lambda_{c,i,j} = \sum_{j \neq i} \lambda_{c,j} \cdot P_c(j,i)
\]

(6)

\[
P_{c}(i,j) = \frac{\lambda_{c,i,j}}{\lambda_{c,j}}, \quad j \neq i
\]

\[
F_{c,i}(t) = 1 - \exp(-\lambda_{c,j} \cdot t)
\]

which gives the semi-Markov characterisation of a continuous time Markov chain.

The above probability \( P_c(i,j) \) arises by sampling \( K_c \) independent exponential random variables with parameters \( \{ \lambda_{c,1}, \ldots, \lambda_{c,K_c} \} \) right after entering state \( i \). The next state will be the one corresponding to the minimum of these.
C. Steady state behaviour of the whole system

In this section we consider the system made up of the UPS and the mains (with $C$ components in total), and we derive a formula to evaluate the MTBF of the output compensator voltage $V_{OUT}$.

As $t \to \infty$, the joint steady state probability that component $c$ is found in state $i$, the next state is $j$, and the time spent in state $i$ before the transition is not greater than $x$ is

$$\frac{P(i, j)}{\mu_i} \int_0^x (1 - F_j(t)) dt$$  \hspace{1cm} (7)

(see [5], Section 5.7).

Now let $F$ be the set of those states $s$ of the system that cause a load supply interruption (failure states) and let $B$ be the set of all the other states (functioning states). This derived two-state process is obtained via the structure function induced by the failure trees in Figure 2, applied to the multivariate process of the components, and is obviously strictly stationary. Therefore the MTBF can be calculated as the reciprocal of the steady state frequency of $F$ (see [7], Section 2.5):

$$Fr(F) = \lim_{t \to \infty} \lim_{\Delta \to 0} \frac{\Pr(S(t) \in F \cap S(t + \Delta) \in B)}{\Delta}$$

$$= \sum_{s \in F} \Pr(s) \left( \sum_{s \in B} \lim_{t \to \infty} \lim_{\Delta \to 0} \frac{\Pr(S(t + \Delta) = s | S(t) = s)}{\Delta} \right)$$  \hspace{1cm} (8)

where $\Pr(s)$ is the steady state probability that the system is found in state $s$. The probability of having two or more components changing state in $[t, t+\Delta]$ is negligible and so only the transition rates involving one component do not vanish in the previous formula. Denoting by $s(x')$ the state $s$ where only component $c$ changes state from $x$ to $x'$, we get:

$$Fr(F) = \sum_{s \in F} \Pr(s) \left( \sum_{c=1}^C \sum_{x} \lim_{t \to \infty} \lim_{\Delta \to 0} \frac{\Pr(X_c(t + \Delta) = x' | X_c(t) = x)}{\Delta} \right)$$  \hspace{1cm} (9)

Each term in the above sum between parenthesis, by theorem 5.17 of [5], is found to be equal to:

$$\frac{P(x, x')}{\mu_{x', x}} \Pr(x) = \frac{P(x, x')}{\mu_{x', x}}$$  \hspace{1cm} (10)

which finally gives

$$Fr(F) = \sum_{s \in F} \Pr(s) \left( \sum_{c=1}^C \sum_{x} \frac{P(x, x')}{\mu_{x', x}} \right) = \frac{1}{MTBF}$$  \hspace{1cm} (11)

In case the components fail and are repaired independently, the steady state probability of system states can be calculated as:

$$\Pr(s) = \prod_{c} \Pr(x_c) \hspace{1cm} s = (x_1, ..., x_C)$$  \hspace{1cm} (12)

so that (11) is easily computed.
IV. RELIABILITY PERFORMANCE ANALYSIS

A. Analytical calculation of reliability indices

By (12) and (2), the $MTBF$ (11) of the system is completely determined by the transition matrices of the embedded Markov chain $P_c$ and by the mean durations ($\mu_{c,i}$, $\forall i$) of all the components. Then the $MTBF$ remains unchanged as long as the same is true for these quantities, regardless of how the underlying duration distributions $F_{c,ij}$ are chosen. Hence the only entertained assumptions are those of semi-Markovian behaviour and of independence of the components.

Often the transition probabilities cannot be computed straightforwardly from the data, but the frequency of departure from a state $i$ to one of the other states can be:

$$F_{c,i}(j) = \lim_{t \to \infty, \Delta \to 0} \Pr(X_c(t) = i, X_c(t+\Delta) = j), \quad j \neq i$$  \hspace{1cm} (13)

Again by theorem 5.17 of [5],

$$F_{c,i}(j) = \frac{P_c(i,j)}{\mu_{c,i}}, \quad j \neq i$$  \hspace{1cm} (14)

which gives $P_c(i,j) = F_{c,i}(j)/F_{c,i}(i)$, for $j \neq i$.

In our case, for each component $c$, there is one state of correct operation (state 1), and the other states, labelled from 2 to $K_c$, are those of incorrect operating conditions. Figure 3 displays the state-space diagram of the generic component along with the frequencies.

\[ \text{Figure 3} \quad \text{Model for the generic component} \ c \ \text{used to perform reliability analysis of a compensation device.} \]

The diagram shows that each state in the embedded Markov chain has period 2, because it switches between state 1 and one of the other states. It is easy to show that the subchain where the failure states are visited is irreducible and positive recurrent, which gives an irreducible and positive recurrent $P_c$ as stated in section III.B.

From this discussion and from equation (3), letting $F_{c,i}(j) = F_{c,i}(j)$ (for $j \geq 2$) and $F_{c,i}(1) = \sum_{j=2}^{K_c} F_{c,i}(j)$, the quantities required for (11) are then found as

$$P_c(i,j) = \frac{F_{c,i}(j)}{F_{c,i}(1)} \quad j = 2, \ldots, K_c$$  \hspace{1cm} (15.1)

$$P_c(i,1) = 1 \quad j = 2, \ldots, K_c$$  \hspace{1cm} (15.2)

$$F_{c,i}(j) = F_{c,i}(1) \cdot \mu_{c,j} \quad i \geq 2$$  \hspace{1cm} (15.3)
The reliability information about the components, shown in table 2, has been obtained from the measurements carried out on the mains in some countries of Europe [8] and from experimental results on compensation devices obtained by a UPS manufacturer [9].

<table>
<thead>
<tr>
<th>Component</th>
<th>Failure</th>
<th>Mean restoration time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mains</td>
<td>$F_{t,M_a}(2) = 6.772 \cdot 10^{-3}$ $h^{-1}$</td>
<td>$\mu_{M_a,2} = 0.22s$</td>
</tr>
<tr>
<td></td>
<td>$F_{t,M_a}(3) = 5.016 \cdot 10^{-3}$ $h^{-1}$</td>
<td>$\mu_{M_a,3} = 1.54s$</td>
</tr>
<tr>
<td>Static transfer switch</td>
<td>$F_{t,SW}(2) = F_{t,SW}(3) = 0.2/{1250 \cdot 10^3} h^{-1}$</td>
<td>$\mu_{SW,2} = \mu_{SW,3} = \mu_{SW,4} = \mu_{SW,5} = \mu_{SW,6} = 10h$</td>
</tr>
<tr>
<td>MTBF$\text{power}=1250kh$</td>
<td>$F_{t,SW}(4) = F_{t,SW}(5) = 0.8/{1250 \cdot 10^3} h^{-1}$</td>
<td></td>
</tr>
<tr>
<td>MTBF$\text{control}=600kh$</td>
<td>$F_{t,SW}(6) = 1/{600 \cdot 10^3} h^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Converters (rectifier, inverter)</td>
<td>$F_{t,R}(2) = F_{t,R}(3) = 0.2/{130 \cdot 10^3} h^{-1}$</td>
<td>$\mu_{R,2} = \mu_{R,3} = \mu_{R,4} = \mu_{R,5} = \mu_{R,6} = \mu_{R,7} = 10h$</td>
</tr>
<tr>
<td>MTBF$\text{power}=130kh$</td>
<td>$F_{t,R}(3) = F_{t,R}(4) = 0.8/{130 \cdot 10^3} h^{-1}$</td>
<td></td>
</tr>
<tr>
<td>MTBF$\text{control}=600kh$</td>
<td>$F_{t,R}(4) = F_{t,R}(5) = 1/{600 \cdot 10^3} h^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Converters (rectifier, inverter)</td>
<td>$F_{t,E}(2) = F_{t,E}(3) = 0.5/{100 \cdot 10^3} h^{-1}$</td>
<td>$\mu_{E,2} = \mu_{E,3} = \mu_{E,4} = \mu_{E,5} = \mu_{E,6} = \mu_{E,7} = 10h$</td>
</tr>
<tr>
<td>Energy storage unit</td>
<td>$F_{t,E}(3) = F_{t,E}(4) = 1/{100 \cdot 10^3} h^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Reliability information about the components of the system.

The failure trees shown above require that the mains voltage should be classified according to three operational modes: between $0.9$ and $1.1V_n$ (mode $Ma_1$), between $0.7$ and $0.9V_n$ (mode $Ma_2$) and below $0.7V_n$ (mode $Ma_3$).

The frequencies of entering the mains states can be computed simply from the statistics mentioned above, reporting the number of voltage sags in a measurement site within a specified period.

The frequencies of visits to the incorrect operating states of the other components can be computed, taking power circuit failure as example, as follows:

$$Pr_r(i) = \lim_{i \to -\Delta} \lim_{\Delta \to 0} \frac{Pr(X_r(t) = 1 \cap X_r(t+\Delta) = i)}{\Delta}$$

$$= \frac{1}{MTBF_{power}} \lim_{n \to -c} \lim_{c \to F_{power}} \{J_{c,n} = d | J_{c,n} \in F_{power}\} \quad i > 1$$

where $F_{power}$ is the set of the failure states for the power circuit of the component $c$. The limit in (16) always exists because it is referred to a subchain of failure states, which is aperiodic, unlike the whole embedded chain of the component states. The manufacturer assumes that 20% of the power circuit failures of the semiconductor devices are short circuits and the remaining 80% are open circuits.
The $MTBF$ of the output compensator voltage $V_{OUT}$, named $MTBF_{OUT}$, is equal to $2.6996 \cdot 10^3 h$. The $MTTR$ of the same voltage, named $MTTR_{OUT}$, is given by

$$MTTR_{OUT} = \left( \sum_{x \in x} Pr(x) \right) \cdot MTBF_{OUT} = 4.4996h$$

(17)

B. Calculation of reliability indices through simulation

In order to check the analytical results, the reliability indices have been calculated through a Monte Carlo simulation. The lifetimes of the components have been represented by exponential distributions, and the repair times by Weibull distributions. Ten independent life histories (with 3000 $V_{OUT}$ failures each) were simulated and the following results have been obtained:

$$MTBF_{MonteCarlo} = 2.6958 \cdot 10^3 h \quad \left( \sigma_{MTBF_{out}} = 4.41 \cdot 10^{-1}h \right)$$

$$MTTR_{MonteCarlo} = 4.5087h \quad \left( \sigma_{MTTR_{out}} = 0.0845h \right)$$

which confirm the validity of the analytical model. The computational effort required to implement the Monte Carlo method is justified if more detailed information than the mean values $MTBF$ and $MTTR$ are required: for example in figure 4 the duration distribution of load supply interruption is represented, which might be required for outage cost calculation.

![Figure 4](image_url)  

Duration distribution function of load supply interruption.

C. Effect of mechanical bypass

Often in compensation devices there is also a mechanical bypass which is closed quickly by the user during an out-of-limit condition of voltage $V_{OUT}$. So during the restoration time the load is supplied directly by the mains and the duration of the load voltage outage shortens and becomes unimportant in cost evaluation.

This component cannot be easily incorporated into the semi-Markov model, because it changes state depending on the duration of the out-of-limit condition and so the stochastic independence among all the components does not hold. Therefore we provide an approximated formula for the calculation of the frequency of load voltage failures. To this purpose, it is convenient to consider the frequency defined in term of counting processes. Let $N(t)$ be the number of load voltage failures observed up to time $t$ in the system with the mechanical bypass. This is the sum of the number of failures observed when the bypass is open plus the number of those observed when it is closed: $N(t) = N(t) + N_{m}(t)$. The random variable $N(t)$ equals the number of failures that we would have without the bypass, whereas $N_{m}(t)$ is the number of
mains sags during bypass closures up to time $t$. Denote also by $F_b$ the set of failure states for the system with mechanical bypass and let $I_b(t)$ be the indicator function of the bypass closure. Then we have

$$
Fr(F_b) = \lim_{\Delta \to 0} \lim_{t \to \infty} E\left[\frac{N(t+\Delta)-N(t)}{\Delta} + \frac{N_m(t+\Delta)-N_m(t)}{\Delta}\right]
$$

$$
= Fr(F) + \lim_{\Delta \to 0} \lim_{t \to \infty} E\left[\frac{N_m(t+\Delta)-N_m(t)}{\Delta} \mid I_b(t)=1\right] \cdot Pr(I_b(t)=1)
$$

(18)

The second term in the last line of (18) is obtained by expanding the expected value $E\left[N_m(t+\Delta)-N_m(t)\right]/\Delta$ as sum of conditional expected values and noting that, conditional on the mechanical bypass being open ($I_b(t)=0$), no mains sags are counted by $N_m(t)$, so that $N_m(t+\Delta)-N_m(t)$ is zero in this situation. Instead conditional on $I_b(t)=1$, the expected value of $\left(N_m(t+\Delta)-N_m(t)\right)/\Delta$ tends to the frequency $Fr(M)$ of the mains sags. As regards $\lim_{t \to \infty} Pr(I_b(t)=1)$, this is approximated from above by the probability $Pr(F)$ that the system without mechanical bypass is in a failure state, because a closed bypass implies that a failure state has been entered. This probability can also be expressed as $MTTR_{OUT}/MTBF_{OUT}$. Then we have

$$
Fr(F_b) = Fr(F) + Fr(M) \cdot Pr(I_b(t)=1) 
\leq \frac{1}{MTBF_{OUT}} \left(1 + MTTR_{OUT}/Fr(M)\right)
$$

(19)

So the mean time between two load voltage drops, named $MTBF_L$, is approximately equal to $2.5636 \cdot 10^3$ h.

The calculation of this reliability index through a Monte Carlo simulation, assuming a time equal to 0.5 hour to close the mechanical bypass, has given $2.5988 \cdot 10^3$ h ($\sigma_{MTBF_L} = 6.93 \cdot 10^3$ h), confirming that the approximation is good.

V. CONCLUSIONS

Semi-Markov processes can be effectively employed for the joint dynamic modelling of compensation devices and mains sags. For the calculation of the $MTBF$ and the $MTTR$ of the load voltage (which is the result of the combination of simple states of the compensation device components and the mains) no assumptions are required on the form of the duration distributions, as the only necessary quantities are the mean duration of each state and the transition probabilities between states. These probabilities are easily obtainable because they are linked to the frequencies and the $MTBF$s of the different modes of failure as shown in Table 2.

The $MTBF$ of the load voltage, in the case that a mechanical bypass switch is present, can be quickly evaluated via a simple approximation, without setting up a more complicated model for its inclusion as a new component. Monte Carlo simulations confirm that both the exact and the approximate formulae given here are valid.
References:


Vitae:

Antonio Pievatolo, received the M.S. degree in Statistics from the University of Padua, Italy, in 1992, and the Doctorate in Statistics from the same university in 1998. He is junior researcher at C.N.R.-I.M.A.T.I. (Consiglio Nazionale delle Ricerche – Istituto per la Matematica Applicata e le Tecnologie Informatiche) since 1997. His research interests include general statistical modelling, reliability, and Markov chain Monte Carlo methods.

Ivan Valadè, received the M.S. degree in Electrical Engineering from the Politecnico di Milano, Italy, in 1999. He is now working toward the Ph.D. degree at Dipartimento di Elettrontecnica of the Politecnico di Milano. His research interests are power electronics and power quality.