Homework 3 - due by 5/7/2025

1. You are going to make a bet about Uruguay winning/losing the 2028 Copa America (football). You will be offered an amount (three choices below) in the case of a defeat and you are asked what you will be willing to offer in case of a victory. You have to compute the expected gains of the two offers and make them equal so that for you it should be equivalent to bet either on the defeat or the victory. This is a method, based on bets and presented in many Bayesian textbooks on Decision Analysis, to assess the probability of an event (here "victory by Uruguay"). I would like you to compute such probability under the three scenarios below and comment on the findings (e.g. if you get different values).

   (a) You are offered $1,000$ Uruguayan pesos

   (b) You are offered $10,000$ Uruguayan pesos

   (c) You are offered $1,000,000$ Uruguayan pesos

2. Consider the following data (naively created by the teacher) about the repayment of a mortgage by people. Data are about gender (M or F), house of property (Y or N), living with a partner (Y or N) and the interest is in the payment back of the mortgage (Y or N).

Paid	Gender	House	Partner
Y	M	Y	Y
Y	M	Y	N
Y	M	Y	N
Y	M	N	Y
Y	F	Y	Y
Y	F	Y	Y
Y	F	N	N
Y	F	N	Y
Y	F	N	Y
N	M	Y	N
N	M	Y	Y
N	M	N	Y
N	M	N	N
N	F	Y	N
N	F	N	Y

(a) Use the Naive Bayes Classifier to decide if the 15 "people" above will repay the mortgage or not and compare your prediction with the actual value.

(b) Using Naive Bayes, classify these three new people:

Gender	House	Partner
F	Y	Y
M	N	N
M	Y	N

(c) Repeat (a) three times after having randomly changed the outcome (paid Y or N) of two "people" (different each time!) and compare the different classification of the 15 "people" (we are mimicking an attack but without using ARA to find the optimal decision). Please comment.

3. Suppose one value x is generated by an exponential distribution, i.e., $X \sim \mathcal{E}(\theta)$, with $\theta = 1$ or $\theta = 2$. An Attacker A can perform two possible attacks and modify x into y:

   - a_1: $y = x$, i.e. no change
   - a_2: $y = 2x$, i.e. doubling the observed value

   Suppose that the Defender D has observed in the past that A chose a_1 4 times out of 10 when $\theta = 1$ and 6 times out of 10 when $\theta = 2$. After observing $y = 1$, compare the probabilities of $\theta = 1$ and $\theta = 2$ and decide which value of θ is more likely.

4. **(Compulsory for posgrado)** Suppose data are generated by a Gaussian distribution $\mathcal{N}(\theta, 1)$. An expert is providing information telling that $(-0.96, 2.96)$ is a 95% credible interval for θ.

   - Which prior distribution (functional form and numerical parameters) would you choose, based on that information?
   - Have you introduced some features of the prior that the expert had not considered?
   - Could you think of alternative choices of priors (no need to compute numerical parameters, although they would be appreciated)?