Homework 2 - due by 17/6/2025

Consider an attacker A (terrorist group) who will put a bomb either at the bus station B or at the train station T. The defender D (police officers) will dispatch officers either to B or T. Therefore, we can consider the action spaces $\mathcal{A} = \{a_B, a_T\}$ and $\mathcal{D} = \{d_B, d_T\}$, corresponding to the actions by A and D, respectively. The subscripts B and T denote if the bomb (or the police) is in B or in T.

The consequences of an attack/defense are given by the set $\mathcal{S} = \{0, 1\}$, where $S = 1$ corresponds to the occurrence of deaths due to the bombing and $S = 0$ corresponds to the case of no killing.

We are making some assumptions:

- A will put the bomb for sure and they will choose only one location
- D will dispatch officers for sure and they will choose only one location
- We are not going to consider the cost of building and placing the bomb and the cost of the officers since they do not change between B and T
- We make no distinction between 1 and more deaths and we do not consider other consequences like injured people, damages to the stations or disruption of services
- We use d to denote actions/decisions by D and a for A
- We use the subscripts D and A to denote utilities and probabilities for D and A, respectively

We are going to look for the optimal decision d^* for the police under different scenarios.

1. Choose a utility function $u_D(d, a, s)$ for all (d, a, s) and justify your choice
2. Choose density functions $f_D(s|d, a)$ for all consequences s and all pairs (d, a) and justify your choice
3. Choose a density function $\pi_D(a)$ about the possible decision by A and justify your choice. This is the case in which D suppose they have sufficient knowledge about A and do not want to consider A as expected utility maximisers, who are possibly guessing about D's decision

4. Solve the problem of interest, with $\pi_D(a)$ known, i.e. find the optimal decision d_1^*:

$$d^* = \arg\max_{d \in \mathcal{D}} \sum_{a \in \mathcal{A}, s \in \mathcal{S}} u_D(d, a, s) f_D(s|d, a) \pi_D(a)$$

5. Now we consider A as expected utility maximisers and present their optimisation problem, i.e., the search for the optimal attack a^*:

$$a^* = \arg\max_{a \in \mathcal{A}} \sum_{d \in \mathcal{D}, s \in \mathcal{S}} u_A(d, a, s) f_A(s|d, a) \pi_A(d)$$

6. Make the simplifying assumption that $u_A(d, a, s) = -u_D(d, a, s)$

7. Consider a space of random densities $F_A(s|d, a)$ from which you generate, for N (sufficiently large) times, densities $f_A(s|d, a)$. Hint:
   - Consider all possible densities (for D!) $f_D(1|d, a) = P(S = 1|d, a)$ for the four possible combinations of d and a
   - Using R, generate (N times) values

     $f_A^{(i)} = (f_A(1|d_B, a_B), f_A(1|d_B, a_T), f_A(1|d_T, a_B), f_A(1|d_T, a_T))$,

     $i = 1, \ldots, N$, from Beta distributions centered at

     $(f_D(1|d_B, a_B), f_D(1|d_B, a_T), f_D(1|d_T, a_B), f_D(1|d_T, a_T))$, respectively

8. Choose a density function $\pi_A(d)$ about what you think A thinks about the possible decision by D and justify your choice

9. Solve N times the optimisation problem for A in (5) and you will find an empirical distribution about $\pi_D(a)$ that, replaced in (4), will give the optimal decision d_2^*

10. Consider now the space of random densities $\Pi_A(d)$ and generate N times the density $\pi_A^{(i)}(d)$, $i = 1, \ldots, n$, from a Beta density as before, but starting from the density function $\pi_A(d)$ chosen in (8)

11. Solve N times the optimisation problem for A in (5) and you will find an empirical distribution about $\pi_D(a)$ that, replaced in (4), will give the optimal decision d_3^*

12. Report the three optimal decisions (d_1^*, d_2^*, d_3^*)