1

## Capitolo W35
# prontuario: integrali

### Contenuti delle sezioni



27 pagine

## W35 a. integrazione

### W35a.01 schemi di integrazione

In questa sessione, salvo segnalazione contraria, assumiamo:
$a, b, c, \alpha, \beta \in \mathbb{R}$ $\eta \in \mathbb{R} \setminus \{-1\}$ $f(x), g(x) \in$ **FunRtR**.

$$\int \mathrm{d}x\ [g(x)]^\eta\, g'(x) \ = \ \frac{1}{b+1}[g(x)]^{\eta+1} + \mathtt{C} \quad , \quad \int \mathrm{d}x\ \frac{g'(x)}{g(x)} \ = \ \ln|g(x)| + \mathtt{C}$$

$$\int \mathrm{d}x\ \sin[g(x)]\ g'(x) \ = \ -\cos[g(x)] \quad , \quad \int \mathrm{d}x\ \cos[g(x)]\ g'(x) \ = \ \sin[g(x)]$$

$$\int \mathrm{d}x\, \frac{1}{\cos^2[g(x)]}\, g'(x) \ = \ \tan[g(x)] + \mathtt{C} \quad , \quad \int \mathrm{d}x\, \frac{1}{\sin^2[g(x)]}\, g'(x) \ = \ \cot[g(x)] + \mathtt{C}$$

$$\int \mathrm{d}x\, a^{g(x)} \cdot g'(x) \ = \ \frac{a^{g(x)}}{\ln a} + \mathtt{C}$$

### W35a.02 regole di integrazione

$$\int \mathrm{d}x\ [\alpha\, f(x) + \beta\, g(x)] \ = \ \alpha \int \mathrm{d}x\ f(x) + \beta \int \mathrm{d}x\, g(x)$$

$$\int_a^c \mathrm{d}x\, f(x) \ = \ \int_a^b \mathrm{d}x\, f(x) + \int_b^c \mathrm{d}x\, f(x)$$

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## W35 b. antiderivate di integrandi algebrici

### W35b.01 antiderivate di integrandi con $a\,x+b$

$$\int \mathrm{d}x\, a \;=\; a\,x + \mathtt{C} \quad \text{con} \quad a\in\mathbb{R}$$

$$\int \mathrm{d}x\;x^{\alpha} \;=\; \frac{x^{\alpha+1}}{\alpha+1} + \mathtt{C} \quad \text{con} \quad \alpha\in\mathbb{R}\setminus\{-1\}$$

$$\int \mathrm{d}x\,\frac{1}{x} \;=\; \ln|x| + \mathtt{C} \quad , \quad \int \mathrm{d}x\,\frac{1}{x^n} \;=\; -\frac{1}{(n-1)\,x^{n-1}} + \mathtt{C}$$

$$\int \mathrm{d}x\, b^x \;=\; \frac{1}{\ln b} b^x + \mathtt{C} \quad \text{con} \quad b\in(0,1)\cup(1,+\infty) \qquad \text{in partic.} \int \mathrm{d}x\;\mathtt{e}^x \;=\; \mathtt{e}^x + \mathtt{C}$$

$$\int \mathrm{d}x\,(a\,x+b)^n \;=\; \frac{(a\,x+b)^{n+1}}{a\,(n+1)} \quad \text{con} \quad n\neq 1$$

$$\int \frac{\mathrm{d}x}{a\,x+b} \;=\; \frac{1}{a}\,\ln|a\,x+b| \quad , \quad \int \frac{\mathrm{d}x}{(a\,x+b)^n} \;=\; \frac{1}{n\,(n-1)\,(a\,x+b)^{n-1}} \quad \text{con} \quad n\neq 1$$

$$\int \mathrm{d}x\;x\,(a\,x+b)^n \;=\; \frac{1}{a^2}\left(\frac{(a\,x+b)^{n+2}}{n+2} - \frac{b(a\,x+b)^{n+1}}{n+1}\right) \quad \text{con} \quad n\neq -1,-2$$

$$\int \mathrm{d}x\,\frac{x}{a\,x+b} \;=\; \frac{x}{a} + \frac{b}{a^2}\,\ln|a\,x+b|$$

$$\int \mathrm{d}x\,\frac{x}{(a\,x+b)^n} \;=\; \frac{1}{a^2}\left(\frac{b}{(n-1)\,(a\,x+b)^{n-1}} - \frac{1}{(n-2)\,(a\,x+b)^{n-2}}\right) \quad \text{con} \quad n\neq 1$$

$$\int \mathrm{d}x\,\frac{x^2}{a\,x+b} \;=\; \frac{x}{2\,a} + \frac{b\,x}{a^2} + \frac{b^2}{a^3}\,\ln|a\,x+b|$$

$$\int \mathrm{d}x\,\frac{x^n}{a\,x+b} \;=\; \frac{x^n}{na} - \frac{b\,x^{n-1}}{(n-1)\,a^2} + \frac{b^2x^{n-2}}{(n-2)\,a^3} - \cdots$$
$$+(-1)^{n-1}\,\frac{b^{n-1}\,x}{a^n} + (-1)^{n-1}\,\frac{b^{n-1}\,x}{a^n}\,\ln|a\,x+b| \quad \text{per} \quad n=3,4,5,...$$

$$\int \mathrm{d}x\,\frac{x}{(a\,x+b)^2} \;=\; \frac{b}{a^2\,(a\,x+b)} + \frac{1}{a^2}\,\ln\,|a\,x+b|$$

$$\int \mathrm{d}x\;x^2\,(a\,x+b)^n \;=\; \frac{1}{a^3}\left(\frac{(a\,x+b)^{n+3}}{n+3} - \frac{2b(a\,x+b)^{n+2}}{n+2} + \frac{b^2(a\,x+b)^{n+1}}{n+1}\right) \quad \text{con} \quad n\neq -1,-2,-3$$

$$\int \mathrm{d}x\,\frac{x^2}{(a\,x+b)^n} \;=\; \frac{1}{a^3}\left(\frac{b^2}{(n-1)(a\,x+b)^{n-1}} - \frac{2\,b}{(n-2)(a\,x+b)^{n-2}} + \frac{1}{(n-3)(a\,x+b)^{n-3}}\right) \quad \text{con} \quad n\neq 1,2,3$$

$$\int \mathrm{d}x\,\frac{x^2}{a\,x+b} \;=\; \frac{1}{a^3}\left(\frac{(a\,x+b)^2}{2} - 2\,b\,(a\,x+b) + b^2\,\ln|a\,x+b|\right)$$

$$\int \mathrm{d}x\,\frac{x^2}{(a\,x+b)^2} \;=\; \frac{1}{a^3}\left(a\,x+b - 2\,b\,\ln|a\,x+b| - \frac{b^2}{a\,x+b}\right)$$

$$\int \mathrm{d}x\,\frac{x^2}{(a\,x+b)^3} \;=\; \frac{1}{a^3}\left(\ln|a\,x+b| + \frac{2\,b}{a\,x+b} - \frac{b^2}{2(a\,x+b)^2}\right)$$

$$\int \frac{\mathrm{d}x}{x(a\,x+b)} \;=\; \frac{1}{b}\,\ln\left|\frac{a\,x+b}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x^2\,(a\,x+b)} \;=\; \frac{1}{b\,x} + \frac{a}{b^2}\,\ln\left|\frac{a\,x+b}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x^3\,(a\,x+b)} \;=\; \frac{2\,a\,x-b}{2\,b^2\,x^2} - \frac{a^2}{b^3}\,\ln\left|\frac{a\,x+b}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x^n\,(a\,x+b)} \;=\; -\frac{1}{(n-1)\,b\,x^{n-1}} + \frac{a}{(n-2)\,b^2\,x^{n-2}} - \frac{a^2}{(n-3)\,b^3\,x^{n-3}} + \cdots$$
$$+\,(-1)^{n-1}\,\frac{a^{n-2}}{b^{n-1}\,x} + (-1)^n\,\frac{a^{n-1}}{b^n}\,\ln\left|\frac{a\,x+b}{x}\right| \quad \text{per} \quad n=4,5,6,...$$

$$\int \frac{\mathrm{d}x}{x\,(a\,x+b)^2} \;=\; \frac{1}{b\,(a\,x+b)} - \frac{1}{b^2}\,\ln\left|\frac{a\,x+b}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x\,(a\,x+b)^n} \;=\; \frac{1}{b\,(n-1)\,(a\,x+b)^{n-1}} + \frac{1}{b}\int \frac{\mathrm{d}x}{x\,(a\,x+b)^{n-1}} \quad \text{per} \quad n \neq 1$$

$$\int \frac{\mathrm{d}x}{x^2\,(a\,x+b)^2} \;=\; -\frac{2\,a\,x+b}{b^2\,x\,(a\,x+b)} + \frac{2\,a}{b^3}\,\ln\left|\frac{a\,x+b}{x}\right|$$

## W35b.02 antiderivate di integrandi con $\sqrt{a\,x+b}$

$$\int \mathrm{d}x\;\sqrt{a\,x+b} \;=\; \frac{2(a\,x+b)^{3/2}}{3\,a}$$

$$\int \mathrm{d}x\;x\,\sqrt{a\,x+b} \;=\; \frac{2(a\,x+b)^{3/2}}{15\,a^2}\,(3\,a\,x-2\,b)$$

$$\int \mathrm{d}x\;x^2\,\sqrt{a\,x+b} \;=\; \frac{2(a\,x+b)^{3/2}}{105\,a^2}\,(15\,a^2\,x^2-12\,a\,b\,x+8\,b^2)$$

$$\int \mathrm{d}x\;x^n\,\sqrt{a\,x+b} \;=\; \frac{2}{a(2\,n+3)}\left(x^n\,(a\,x+b)^{3/2} - b\,n\int \mathrm{d}x\;x^{n-1}\,\sqrt{a\,x+b}\right)$$

$$\int \mathrm{d}x\,\frac{\sqrt{a\,x+b}}{x} \;=\; 2\,\sqrt{a\,x+b} + b\int \frac{\mathrm{d}x}{x\,\sqrt{a\,x+b}} \qquad V.(1)$$

$$\int \mathrm{d}x\,\frac{\sqrt{a\,x+b}}{x^n} \;=\; \frac{1}{b(n-1)}\left(\frac{(a\,x+b)^{3/2}}{x^{n-1}} + \frac{2\,n-5)\,a}{2}\int \mathrm{d}x\,\sqrt{\frac{a\,x+b}{x^{n-1}}}\right)$$

$$\int \frac{\mathrm{d}x}{\sqrt{a\,x+b}} \;=\; \frac{2\,\sqrt{a\,x+b}}{a}$$

$$\int \mathrm{d}x\,\frac{x}{\sqrt{a\,x+b}} \;=\; \frac{2\,b\,\sqrt{a\,x+b}}{a^2} + \frac{2(a\,x+b)^{3/2}}{3\,a^2}$$

$$\int \mathrm{d}x\,\frac{x^2}{\sqrt{a\,x+b}} \;=\; \frac{2\,b^2\,\sqrt{a\,x+b}}{a^3} - \frac{4\,b(a\,x+b)^{3/2}}{3\,a^3} + \frac{2(a\,x+b)^{5/2}}{5\,a^3}$$

$$\int \mathrm{d}x\,\frac{x^n}{\sqrt{a\,x+b}} \;=\; \frac{2}{a(2\,n+1)}\left(x^n\,\sqrt{a\,x+b} - b\,n\int \mathrm{d}x\,\frac{x^{n-1}}{\sqrt{a\,x+b}}\right)$$

$$(1) \qquad \int \frac{\mathrm{d}x}{x\,\sqrt{a\,x+b}} \;=\; \begin{cases} \frac{1}{\sqrt{b}}\,\ln\left|\frac{\sqrt{a\,x+b}-\sqrt{b}}{\sqrt{a\,x+b}+\sqrt{b}}\right| & \text{sse } b>0 \\ \frac{2}{\sqrt{-b}}\,\arctan\sqrt{\frac{a\,x+b}{-b}} & \text{sse } b<0 \end{cases}$$

$$\int \frac{\mathrm{d}x}{x^n\,\sqrt{a\,x+b}} \;=\; -\frac{\sqrt{a\,x+b}-\sqrt{b}}{\sqrt{a\,x+b}+\sqrt{b}} - \frac{(2\,n-3)\,a}{(2\,n-2)b}\int \frac{\mathrm{d}x}{x^{n-1}\,\sqrt{a\,x+b}} \quad \text{per} \quad n \neq -1$$

$$\int \frac{\mathrm{d}x}{c+\sqrt{a\,x+b}} \;=\; \frac{2}{a}\left(\sqrt{a\,x+b} - c\,\ln\left|c+\sqrt{a\,x+b}\right|\right)$$

$$\int \mathrm{d}x\,\frac{\sqrt{a\,x+b}}{c+\sqrt{a\,x+b}} \;=\; \frac{2}{a}\left(a\,x+b-2\,c\,\sqrt{a\,x+b}+2\,c^2\,\ln\left|c+\sqrt{a\,x+b}\right|\right)$$

$$\int \mathrm{d}x\,\frac{x}{c+\sqrt{a\,x+b}} \;=\; \frac{1}{a^2}\left(2(c^2-b)\,\sqrt{a\,x+b} - c\,(a\,x+b) + \frac{2}{3}\,(a\,x+b)^{3/2} - 2\,c\,(c^2-b)\,\ln\left|c+\sqrt{a\,x+b}\right|\right)$$

$$\int \frac{\mathrm{d}x}{\sqrt{a\,x+b}\,\left(c+\sqrt{a\,x+b}\right)} \;=\; \frac{2}{a}\,\ln\left|c+\sqrt{a\,x+b}\right|$$

$$\int \frac{\mathrm{d}x}{(a\,x+b)\,\left(c+\sqrt{a\,x+b}\right)} \;=\; \frac{2}{a\,c}\,\ln\left|\frac{\sqrt{a\,x+b}}{c+\sqrt{a\,x+b}}\right|$$

$$\int \frac{\mathrm{d}x}{\left(c+\sqrt{a\,x+b}\right)^2} \;=\; \frac{2\,c}{a\left(c+\sqrt{a\,x+b}\right)}+\frac{2}{a}\,\ln\left|c+\sqrt{a\,x+b}\right|$$

$$\int \mathrm{d}x\,\frac{\sqrt{a\,x+b}}{\left(c+\sqrt{a\,x+b}\right)^2} \;=\; \frac{2\sqrt{a\,x+b}}{a}-\frac{2\,c^2}{a\left(c+\sqrt{a\,x+b}\right)}-\frac{4\,c}{a}\,\ln\left|c+\sqrt{a\,x+b}\right|$$

$$\int \mathrm{d}x\,\frac{x}{\left(c+\sqrt{a\,x+b}\right)^2} \;=\; \frac{1}{a^2}\left(-4\,c\,\sqrt{a\,x+b}+a\,x+\frac{2\,c(c^2-b)}{c+\sqrt{a\,x+b}}+2(3\,c^2-b)\,\ln\left|c+\sqrt{a\,x+b}\right|\right)$$

$$\int \frac{\mathrm{d}x}{\sqrt{a\,x+b}\left(c+\sqrt{a\,x+b}\right)^2} \;=\; -\frac{2}{a\left(c+\sqrt{a\,x+b}\right)}$$

$$\int \frac{\mathrm{d}x}{(a\,x+b)\left(c+\sqrt{a\,x+b}\right)^2} \;=\; \frac{1}{a\,c^2}\left(\frac{2\,c}{c+\sqrt{a\,x+b}}+2\,\ln\left|\frac{\sqrt{a\,x+b}}{c+\sqrt{a\,x+b}}\right|\right)$$

## W35**b.03** **antiderivate di integrandi con** $a\,x+b$ **e** $c\,x+d$

Consideriamo i reali $a$, $b$, $c$ e $d$, poniamo $k := a\,d-b\,c$ e supponiamo che tale numero sia diverso da 0; consideriamo inoltre $m = 2, 3, \ldots$ ed $n = 1, 2, 3, \ldots$ .

$$\int \frac{\mathrm{d}x}{(\ x+b)^n\,(c\,x+d)^m} \;=\;$$
$$\frac{1}{k(m-1)}\left[\frac{1}{(a\,x+b)^{n-1}\,(c\,x+d)^{m-1}}+a(m+n-2)\int\frac{\mathrm{d}x}{(\ x+b)^n\,(c\,x+d)^{m-1}}\right]$$

$$\int \frac{\mathrm{d}x}{(a\,x+b)\,(c\,x+d)} \;=\; \frac{1}{k}\,\ln\left|\frac{a\,x+b}{c\,x+d}\right|$$

$$\int \mathrm{d}x\,\frac{x}{(a\,x+b)\,(c\,x+d)} \;=\; -\frac{1}{k}\left(\frac{b}{a}\,\ln|a\,x+b|-\frac{\mathrm{d}}{c}\,\ln|c\,x+d|\right)$$

$$\int \frac{\mathrm{d}x}{(a\,x+b)^2\,(c\,x+d)} \;=\; \frac{1}{k}\left(\frac{1}{a\,x+b}+\frac{c}{k}\,\ln\left|\frac{a\,x+b}{c\,x+d}\right|\right)$$

$$\int \mathrm{d}x\,\frac{x}{(a\,x+b)^2\,(cx+d)} \;=\; \frac{b}{a\,k(a\,x+b)}+\frac{d}{k^2}\,\ln\left|\frac{a\,x+b}{c\,x+d}\right|$$

$$\int \mathrm{d}x\,\frac{x^2}{(a\,x+b)^2\,(c\,x+d)} \;=\; -\frac{b^2}{a^2\,k(a\,x+b)}+\frac{1}{k^2}\left(-\frac{b(k+a\,d)}{a^2}\,\ln|a\,x+b|+\frac{d^2}{c}\,\ln|c\,x+d|\right)$$

$$\int \frac{\mathrm{d}x}{x\,(a\,x+b)\,(c\,x+d)} \;=\; \frac{1}{b\,d}\,\ln|x|-\frac{a}{b\,k}\,\ln|a\,x+b|+\frac{c}{d\,k}\,\ln|c\,x+d|$$

$$\int \mathrm{d}x\,\frac{d}{x^2\,(a\,x+b)^2\,(c\,x+d)} \;=\; -\frac{a^2\,d^2+b^2\,c^2}{b^2\,d^2\,k}\,\ln|x|-\frac{1}{b\,d\,x}+\frac{a^2}{b^2\,k}\,\ln|a\,x+b|+\frac{c^2}{d^2\,k}\,\ln|c\,x+d|$$

$$\int \mathrm{d}x\,\frac{a\,x+b}{c\,x+d} \;=\; \frac{a\,x}{c}-\frac{k}{c^2}\,\ln|c\,x+d|$$

$$\int \mathrm{d}x\, \frac{(a\,x+b)^n}{(c\,x+d)^m} = \frac{1}{k(m-1)} \left( \frac{(a\,x+b)^{n+1}}{(c\,x+d)^{m-1}} + (m-n-2)\,a \int \mathrm{d}x\, \frac{(a\,x+b)^n}{(c\,x+d)^{m-1}} \right)$$
$$= -\frac{1}{(m-n-1)\,c} \left( \frac{(a\,x+b)^n}{(c\,x+d)^{m-1}} - k\,n \int \mathrm{d}x\, \frac{(a\,x+b)^{n-1}}{(c\,x+d)^m} \right)$$

$$\int \mathrm{d}x\, \sqrt{\frac{x+b}{x+d}} = \sqrt{x+b}\,\sqrt{x+d} + (b-d)\,\ln\left(\sqrt{x+d}+\sqrt{x+d}\right)$$

$$\int \mathrm{d}x\, \sqrt{\frac{b-x}{d+x}} = \sqrt{b-x}\,\sqrt{x+d} + (b+d)\,\arcsin\sqrt{\frac{d+x}{b+d}}$$

$$\int \frac{\mathrm{d}x}{(c\,x+d)\,\sqrt{x+b}} = \begin{cases} \frac{1}{\sqrt{-k\,c}}\,\ln\left|\frac{\sqrt{c(a\,x+b)}-\sqrt{-k}}{\sqrt{c(a\,x+b)}+\sqrt{-k}}\right| & \text{se } c>0\ ,\ k<0 \\ \frac{2}{\sqrt{k\,c}}\,\arctan\sqrt{\frac{c(\,x+b)}{k}} & \text{se } c,k>0 \end{cases}$$

## W35**b.04** antiderivate di integrandi con $a^2\,x^2 \pm c^2$

Consideriamo $a$ e $c$ i reali positivi.

$$\int \frac{\mathrm{d}x}{a^2\,x^2+c^2} = \frac{1}{a\,c}\,\arctan\frac{x}{c} =:\ A_1$$

$$\int \frac{\mathrm{d}x}{a^2\,x^2-c^2} = \frac{1}{2\,a\,c}\,\ln\left|\frac{a\,x-c}{a\,x+c}\right| =:\ B_1$$

$$\int \frac{\mathrm{d}x}{(a^2\,x^2+c^2)^2} = \frac{x}{2\,c^2(a^2\,x^2+c^2} + \frac{1}{2\,a\,c^3}\,\arctan\frac{a\,x}{c} =:\ A_2$$

$$\int \frac{\mathrm{d}x}{(a^2\,x^2-c^2)^2} = -\frac{x}{2\,c^2(a^2\,x^2-c^2} - \frac{1}{4\,a\,c^3}\,\ln\left|\frac{a\,x-c}{a\,x+c}\right| =:\ B_2$$

$$A_n := \int \frac{\mathrm{d}x}{(a^2\,x^2+c^2)^n} = \frac{x}{2(n-1)c^2(a^2\,x^2)^{n-1}} + \frac{2\,n-3}{2(n-1)c^2}\,A_{n-1}$$

$$B_n := \int \frac{\mathrm{d}x}{(a^2\,x^2-c^2)^n} = \frac{x}{2(n-1)\,c^2\,(a^2\,x^2)^{n-1}} - \frac{2\,n-3}{2(n-1)c^2}\,B_{n-1}$$

$$\int \mathrm{d}x\ x\,(a^2\,x^2 \pm c^2)^n = \frac{(a^2\,x^2 \pm c^2)^{n+1}}{2(n+1)\,a^2} \quad \text{per} \quad n \neq -1$$

$$\int \mathrm{d}x\, \frac{x}{a^2\,x^2 \pm c^2} = \frac{1}{2\,a^2}\,\ln\left|a^2\,x^2 \pm c^2\right|$$

$$\int \mathrm{d}x\, \frac{x}{(a^2\,x^2 \pm c^2)^n} = \frac{1}{2\,a^2\,(n-1)\,(a^2\,x^2 \pm c^2)^{n-1}} \quad \text{per} \quad n \neq 1$$

$$\int \frac{\mathrm{d}x}{x\,(a^2\,x^2 \pm c^2)} = \pm\frac{1}{2\,c^2}\,\ln\left|\frac{x^2}{a^2\,x^2 \pm c^2}\right|$$

$$\int \frac{\mathrm{d}x}{x^2\,(a^2\,x^2+c^2)} = -\frac{1}{c^2\,x} - \frac{a}{c^3}\,\arctan\frac{a\,x}{c}$$

$$\int \frac{\mathrm{d}x}{x^2\,(a^2\,x^2-c^2)} = \frac{1}{c^2\,x} + \frac{a}{2\,c^3}\,\ln\left|\frac{a\,x-c}{ax+c}\right|$$

$$\int \mathrm{d}x\, \frac{x^2}{a^2\,x^2+c^2} = \frac{x}{a^2} - \frac{c}{a^3}\,\arctan\frac{a\,x}{c}$$

$$\int \mathrm{d}x\, \frac{x^2}{a^2\,x^2-c^2} = \frac{x}{a^2} + \frac{c}{2\,a^3}\,\ln\left|\frac{a\,x-c}{a\,x+c}\right|$$

$$\int dx\, \frac{x^n}{a^2\,x^2 \pm c^2} \;=\; \frac{x^{n-1}}{a^2(n-1)} \mp \frac{c^2}{a^2} \int dx\, \frac{x^{n-2}}{a^2\,x^2 \pm c^2} \qquad \text{per} \quad n \neq 1$$

$$\int dx\, \frac{x^2}{(a^2\,x^2 + c^2)^n} \;=\; -\frac{x}{2(n-1)\,a^2(a^2\,x^2 + c^2)^{n-1}} + \frac{1}{2(n-1)\,a^2}\,A_{n-1} \qquad \text{per} \quad n \neq 1$$

$$\int dx\, \frac{x^2}{(a^2\,x^2 - c^2)^n} \;=\; -\frac{x}{2(n-1)\,a^2(a^2\,x^2 - c^2)^{n-1}} + \frac{1}{2(n-1)\,a^2}\,B_{n-1} \qquad \text{per} \quad n \neq 1$$

$$\int dx\, \frac{x^m}{(a^2\,x^2 \pm c^2)^n} \;=\; \frac{1}{a^2} \int dx\, \frac{x^{m-2}}{(a^2\,x^2 \pm c^2)^{n-1}} \mp \frac{c^2}{a^2} \int dx\, \frac{x^{m-2}}{(a^2\,x^2 \pm c^2)^n}$$

$$\int \frac{dx}{x\,(a^2\,x^2 \pm c^2)^n} \;=\; \pm\frac{1}{2\,c^2(n-1)(a^2\,x^2 \pm c^2)^{n-1}} \pm \frac{1}{c^2} \int \frac{dx}{(a^2\,x^2 \pm c^2)^{n-1}} \qquad \text{per} \quad n \neq 1$$

$$\int \frac{dx}{x^2\,(a^2\,x^2 \pm c^2)^n} \;=\; \pm\frac{1}{c^2} \int \frac{dx}{x^2\,(a^2\,x^2 \pm c^2)^{n-1}} \mp \frac{a^2}{c^2} \int \frac{dx}{(a^2\,x^2 \pm c^2)^n}$$

$$\int \frac{dx}{x^m\,(a^2\,x^2 \pm c^2)^n} \;=\; \pm\frac{1}{c^2} \int \frac{dx}{x^m\,(a^2\,x^2 \pm c^2)^{n-1}} \mp \frac{a^2}{c^2} \int \frac{dx}{x^{m-2}\,(a^2\,x^2 \pm c^2)^n}$$

$$\int \frac{dx}{(p\,x + q)\,(a^2\,x^2 + c^2)} \;=\; \frac{1}{a^2\,q^2 + c^2\,p^2} \left( \frac{p}{2}\,\ln \frac{(p\,x+q)^2}{a^2\,x^2 + c^2} + \frac{a\,q}{2\,c}\,\arctan \frac{a\,x}{c} \right)$$

$$\int \frac{dx}{(p\,x + q)\,(a^2\,x^2 - c^2)} \;=\; \frac{1}{a^2\,q^2 - c^2\,p^2} \left( \frac{p}{2}\,\ln \frac{(p\,x+q)^2}{|a^2\,x^2 - c^2|} + \frac{a\,q}{2\,c}\,\ln \left| \frac{a\,x - c}{a\,x + c} \right| \right)$$

$$\int dx\, \frac{x}{(p\,x + q)\,(a^2\,x^2 + c^2)} \;=\; \frac{1}{a^2\,q^2 + c^2\,p^2} \left( -\frac{q}{2}\,\ln \frac{(p\,x+q)^2}{a^2\,x^2 + c^2} + \frac{c\,p}{a}\,\arctan \frac{a\,x}{c} \right)$$

$$\int dx\, \frac{x}{(p\,x + q)\,(a^2\,x^2 - c^2)} \;=\; \frac{1}{a^2\,q^2 - c^2\,p^2} \left( -\frac{q}{2}\,\ln \frac{(p\,x+q)^2}{|a^2\,x^2 - c^2|} - \frac{c\,p}{2\,a}\,\ln \left| \frac{a\,x - c}{a\,x + c} \right| \right)$$

$$\int dx\, \frac{x^2}{(p\,x + q)\,(a^2\,x^2 + c^2)} \;=\; \frac{1}{a^2\,q^2 + c^2\,p^2} \left( \frac{q^2}{p}\,\ln|p\,x+q| + \frac{c^2\,p}{2\,a^2}\,\ln(a^2\,x^2 + c^2) - \frac{c\,q}{a}\,\arctan \frac{a\,x}{c} \right)$$

$$\int dx\, \frac{x^2}{(p\,x + q)\,(a^2\,x^2 - c^2)} \;=\; \frac{1}{a^2\,q^2 - c^2\,p^2} \left( \frac{q^2}{p}\,\ln|p\,x+q| - \frac{c^2\,p}{2\,a^2}\,\ln\left|a^2\,x^2 - c^2\right| + \frac{c\,q}{2\,a}\,\ln \left| \frac{a\,x - c}{a\,x + c} \right| \right)$$

## W35**b.05** **antiderivate di integrandi con $\sqrt{a^2\,x^2 \pm c^2}$**

$$\int dx\, \sqrt{a^2\,x^2 \pm c^2} \;=\; \frac{1}{2}\,x\,\sqrt{a^2\,x^2 \pm c^2} \pm \frac{c^2}{2\,a}\,\ln\left|a\,x + \sqrt{a^2\,x^2 \pm c^2}\right|$$

$$\int \frac{dx}{\sqrt{a^2\,x^2 \pm c^2}} \;=\; \frac{1}{a}\,\ln\left|a\,x + \sqrt{a^2\,x^2 \pm c^2}\right|$$

$$\int dx\, \frac{x}{\sqrt{a^2\,x^2 \pm c^2}} \;=\; \frac{1}{a^2}\,\sqrt{a^2\,x^2 \pm c^2}$$

$$\int \frac{dx}{x\,\sqrt{a^2\,x^2 + c^2}} \;=\; -\frac{1}{c}\,\ln\left| \frac{\sqrt{a^2\,x^2 + c^2} + c}{x} \right|$$

$$\int \frac{dx}{x\,\sqrt{a^2\,x^2 - c^2}} \;=\; \frac{1}{c}\,\arctan \frac{\sqrt{a^2\,x^2 + c^2}}{c} \qquad \left[ = \frac{1}{c}\,\arccos \frac{c}{a\,x} \quad \text{se} \quad x > 0 \right]$$

$$\int dx\; x\,\sqrt{a^2\,x^2 \pm c^2} \;=\; \frac{1}{3\,a^2}\,(a^2\,x^2 \pm c^2)^{3/2}$$

$$(1)\quad \int \mathrm{d}x\ x^2\sqrt{a^2x^2\pm c^2} = \frac{x}{4a^2}(a^2x^2\pm c^2)^{3/2}\mp\frac{c^2x}{8a^2}\sqrt{a^2x^2\pm c^2}-\frac{c^4}{8a^3}\ln\left|ax+\sqrt{a^2x^2\pm c^2}\right|$$

$$\int \mathrm{d}x\,\frac{\sqrt{a^2x^2+c^2}}{x} = \sqrt{a^2x^2+c^2}-c\ln\left|\frac{a^2x^2+c^2}{x}\right|$$

$$\int \mathrm{d}x\,\frac{\sqrt{a^2x^2-c^2}}{x} = \sqrt{a^2x^2-c^2}-c\arctan\frac{\sqrt{a^2x^2-c^2}}{c}$$

$$\int \frac{\mathrm{d}x}{x^2\sqrt{a^2x^2\pm c^2}} = \mp\frac{\sqrt{a^2x^2\pm c^2}}{c^2x}$$

$$\int \mathrm{d}x\,\frac{x^n}{\sqrt{a^2x^2\pm c^2}} = \frac{x^{n-1}\sqrt{a^2x^2\pm c^2}}{n\,a^2}\mp\frac{(n-1)c^2}{n\,a^2}\int \mathrm{d}x\,\frac{x^{n-2}}{\sqrt{a^2x^2\pm c^2}}\quad \text{per}\quad n=1,2,3,...$$

$$\int \mathrm{d}x\,x^n\sqrt{a^2x^2\pm c^2} = \frac{x^{n-1}(a^2x^2\pm c^2)^{3/2}}{(n+2)\,a^2}\mp\frac{(n-1)c^2}{(n+2)\,a^2}\int \mathrm{d}x\,x^{n-2}\sqrt{a^2x^2\pm c^2}\quad \text{per}\quad n=1,2,3,...$$

$$\int \mathrm{d}x\,\frac{\sqrt{a^2x^2\pm c^2}}{x^n} = \mp\frac{(a^2x^2\pm c^2)^{3/2}}{(n-1)\,c^2x^{n-1}}\mp\frac{(n-4)\,a^2}{(n-1)\,c^2}\int \mathrm{d}x\,\frac{\sqrt{a^2x^2\pm c^2}}{x^{n-2}}\quad \text{per}\quad n=1,2,3,...$$

$$\int \frac{\mathrm{d}x}{x^n\sqrt{a^2x^2\pm c^2}} = \mp\frac{\sqrt{(a^2x^2\pm c^2)}}{(n-1)\,c^2x^{n-1}}\mp\frac{(n-2)\,a^2}{(n-1)\,c^2}\int \frac{\mathrm{d}x}{x^{n-2}\sqrt{a^2x^2\pm c^2}}\quad \text{per}\quad n=1,2,3,...$$

$$\int \frac{\mathrm{d}x}{(x-b)\sqrt{x^2-b^2}} = -\frac{1}{b}\sqrt{\frac{x+b}{x-b}}\quad \text{per}\quad b<0\ \wedge\ b>0$$

$$\int \frac{\mathrm{d}x}{(x-b)\sqrt{p\,x^2+q}} = \left\lfloor t:=\frac{1}{x-b}\right\rfloor = -\mathrm{sign}(b)\int \frac{\mathrm{d}t}{\sqrt{(p\,b^2+q)t^2+2\,p\,b\,t+p}}\quad \text{per}\quad b\in\mathbb{R}$$

$$\int \frac{\mathrm{d}x}{(x-b)^n\sqrt{p\,x^2+q}} = \left\lfloor t:=\frac{1}{x-b}\right\rfloor = -\mathrm{sign}(b)\int \mathrm{d}t\,\frac{t^{n-1}}{\sqrt{(p\,b^2+q)t^2+2\,p\,b\,t+p}}\quad \text{per}\quad b\in\mathbb{R}$$

$$\int \mathrm{d}x\,(a^2x^2\pm c^2)^{3/2} = \frac{x}{4}(a^2x^2\pm c^2)^{3/2}\pm\frac{3c^2x}{8}\sqrt{a^2x^2\pm c^2}+\frac{3c^4}{8a}\ln\left|ax+\sqrt{a^2x^2\pm c^2}\right|$$

$$\int \mathrm{d}x\ x\,(a^2x^2\pm c^2)^{3/2} = \frac{1}{5a^2}(a^2x^2\pm c^2)^{5/2}$$

$$\int \mathrm{d}x\ x^2(a^2x^2\pm c^2)^{3/2} = \frac{x^3}{6}(a^2x^2\pm c^2)^{3/2}\pm\frac{c^2}{2}\int \mathrm{d}x\ x^2\sqrt{a^2x^2\pm c^2}$$

$$\int \mathrm{d}x\ x^3(a^2x^2\pm c^2)^{3/2} = \frac{1}{7a^4}(a^2x^2\pm c^2)^{7/2}\mp\frac{c^2}{5a^4}(a^2x^2\pm c^2)^{5/2}$$

$$\int \mathrm{d}x\,\frac{(a^2x^2+c^2)^{3/2}}{x} = \frac{1}{3}(a^2x^2+c^2)^{3/2}+c^2\sqrt{a^2x^2+c^2}-c^3\ln\left|\frac{c+\sqrt{a^2x^2+c^2}}{x}\right|$$

$$\int \mathrm{d}x\,\frac{(a^2x^2-c^2)^{3/2}}{x} = \frac{1}{3}(a^2x^2-c^2)^{3/2}-c^2\sqrt{a^2x^2-c^2}+c^3\arctan\frac{\sqrt{a^2x^2-c^2}}{c}$$

$$\int \frac{\mathrm{d}x}{(a^2x^2\pm c^2)^{3/2}} = \pm\frac{x}{c^2\sqrt{a^2x^2\pm c^2}}$$

$$\int \mathrm{d}x\,\frac{x}{(a^2x^2\pm c^2)^{3/2}} = -\frac{1}{a^2\sqrt{a^2x^2\pm c^2}}$$

$$\int \mathrm{d}x\,\frac{x^2}{(a^2x^2\pm c^2)^{3/2}} = -\frac{x}{a^2\sqrt{a^2x^2\pm c^2}}+\frac{1}{a^3}\ln\left|ax+\sqrt{a^2x^2\pm c^2}\right|$$

$$\int \mathrm{d}x\,\frac{x^3}{(a^2x^2\pm c^2)^{3/2}} = \pm\frac{c^2}{a^4\sqrt{a^2x^2\pm c^2}}+\frac{1}{a^4}\sqrt{a^2x^2\pm c^2}$$

$$\int \frac{\mathrm{d}x}{x\,(a^2\,x^2+c^2)^{3/2}} \;=\; \frac{1}{c^2\,\sqrt{a^2\,x^2+c^2}} - \frac{1}{c^3}\,\ln\left|\frac{c+\sqrt{a^2\,x^2+c^2}}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x\,(a^2\,x^2-c^2)^{3/2}} \;=\; -\frac{1}{c^2\,\sqrt{a^2\,x^2+c^2}} - \frac{1}{c^3}\,\arctan\frac{\sqrt{a^2\,x^2-c^2}}{c}$$

$$\int \frac{\mathrm{d}x}{x^2\,(a^2\,x^2\pm c^2)^{3/2}} \;=\; -\frac{1}{c^4}\left(\frac{\sqrt{a^2\,x^2\pm c^2}}{x}+\frac{a^2\,x}{\sqrt{a^2\,x^2\pm c^2}}\right)$$

$$\int \frac{\mathrm{d}x}{x^3\,(a^2\,x^2+c^2)^{3/2}} \;=\; -\frac{1}{2\,c^2}\left(\frac{1}{x^2\,\sqrt{a^2\,x^2+c^2}}+\frac{3\,a^2}{c^2\,\sqrt{a^2\,x^2+c^2}}+\frac{3\,a^2}{c^3}\,\ln\left|\frac{\sqrt{a^2\,x^2\pm c^2}-c}{x}\right|\right)$$

$$\int \frac{\mathrm{d}x}{x^3\,(a^2\,x^2-c^2)^{3/2}} \;=\; \frac{1}{2\,c^2}\left(\frac{1}{x^2\,\sqrt{a^2\,x^2-c^2}}-\frac{3\,a^2}{c^2\,\sqrt{a^2\,x^2-c^2}}-\frac{3\,a^2}{c^3}\,\arctan\frac{\sqrt{a^2\,x^2-c^2}}{c}\right)$$

## W35b.06 antiderivate di integrandi con $c^2-a^2\,x^2$ per $a,c>0$

$$\int \frac{\mathrm{d}x}{c^2-a^2\,x^2} \;=\; \frac{1}{2\,a\,c}\,\ln\left|\frac{c+a\,x}{c-a\,x}\right| \;=:\; \mathsf{C}_1$$

$$\int \frac{\mathrm{d}x}{(c^2-a^2\,x^2)^2} \;=\; \frac{x}{2\,c^2(c^2-a^2\,x^2)}+\frac{1}{4\,a\,c^3}\,\ln\left|\frac{c+a\,x}{c-a\,x}\right| \;=:\; \mathsf{C}_2$$

(1) $$\int \frac{\mathrm{d}x}{(c^2-a^2\,x^2)^n} \;=\; \frac{x}{2(n-1)\,c^2(c^2-a^2\,x^2)^{n-1}}+\frac{2\,n-3}{2(n-1)c^2}\,\mathsf{C}_{n-1} \;=:\; \mathsf{C}_n$$

$$\int \mathrm{d}x\;x\,(c^2-a^2\,x^2)^n \;=\; -\frac{(c^2-a^2\,x^2)^{n+1}}{2(n+1)a^2} \quad \text{per} \quad n\neq-1$$

$$\int \mathrm{d}x\,\frac{x}{c^2-a^2\,x^2} \;=\; -\frac{1}{2\,a^2}\,\ln\left|c^2-a^2\,x^2\right|$$

$$\int \mathrm{d}x\,\frac{x}{(c^2-a^2\,x^2)^n} \;=\; -\frac{1}{2\,a^2(n-1)\,(c^2-a^2\,x^2)^{n-1}} \quad \text{per} \quad n\neq-1$$

$$\int \frac{\mathrm{d}x}{x(c^2-a^2\,x^2)} \;=\; -\frac{1}{2\,c^2}\,\ln\left|\frac{x^2}{c^2-a^2\,x^2}\right|$$

(2) $$\int \frac{\mathrm{d}x}{x^2(c^2-a^2\,x^2)} \;=\; -\frac{1}{c^2\,x}+\frac{a}{2\,c^3}\,\ln\left|\frac{c+a\,x}{c-a\,x}\right|$$

$$\int \mathrm{d}x\,\frac{x^2}{c^2-a^2\,x^2} \;=\; -\frac{x}{a^2}+\frac{c}{2\,a^3}\,\ln\left|\frac{c+a\,x}{c-a\,x}\right|$$

$$\int \mathrm{d}x\,\frac{x^n}{c^2-a^2\,x^2} \;=\; -\frac{x^{n-1}}{a^2(n-1)}+\frac{c^2}{a^2}\int \mathrm{d}x\,\frac{x^{n-2}}{c^2-a^2\,x^2}$$

$$\int \mathrm{d}x\,\frac{x^2}{(c^2-a^2\,x^2)^n} \;=\; \frac{x}{2(n-1)\,a^2\,(c^2-a^2\,x^2)^{n-1}}-\frac{1}{2(n-1)\,a^2}\,\mathsf{C}_{n-1}$$

$$\int \mathrm{d}x\,\frac{x^m}{(c^2-a^2\,x^2)^n} \;=\; -\frac{1}{a^2}\int \mathrm{d}x\,\frac{x^{m-2}}{(c^2-a^2\,x^2)^{n-1}}+\frac{c^2}{a^2}\int \mathrm{d}x\,\frac{x^{m-2}}{(c^2-a^2\,x^2)^n}$$

$$\int \frac{\mathrm{d}x}{x\,(c^2-a^2\,x^2)^n} \;=\; \frac{1}{2(n-1)\,c^2\,(c^2-a^2\,x^2)^{n-1}}+\frac{1}{c^2}\int \frac{\mathrm{d}x}{x^2\,(c^2-a^2\,x^2)^{n-1}} \quad \text{per} \quad n\neq1$$

$$\int \frac{\mathrm{d}x}{x^2\,(c^2-a^2\,x^2)^n} = \frac{1}{c^2}\int\frac{\mathrm{d}x}{x^2\,(c^2-a^2\,x^2)^{n-1}}+\frac{a^2}{c^2}\int\frac{\mathrm{d}x}{(c^2-a^2\,x^2)^n}$$

$$\int \frac{\mathrm{d}x}{x^m\,(c^2-a^2\,x^2)^n} = \frac{1}{c^2}\int\frac{\mathrm{d}x}{x^m\,(c^2-a^2\,x^2)^{n-1}}+\frac{a^2}{c^2}\int\frac{\mathrm{d}x}{x^{m-2}\,(c^2-a^2\,x^2)^n}$$

$$\int \frac{\mathrm{d}x}{(p\,x+q)\,(c^2-a^2\,x^2)} = \frac{1}{c^2\,p^2-a^2\,q^2}\left(\frac{p}{2}\ln\frac{(p\,x+q)^2}{|c^2-a^2\,x^2|}+\frac{a\,q}{2\,c}\ln\left|\frac{c-a\,x}{c+a\,x}\right|\right)$$

$$\int \mathrm{d}x\,\frac{x}{(p\,x+q)\,(c^2-a^2\,x^2)} = \frac{1}{c^2\,p^2-a^2\,q^2}\left(-\frac{q}{2}\ln\frac{(p\,x+q)^2}{|c^2-a^2\,x^2|}-\frac{c\,p}{2\,a}\ln\left|\frac{c-a\,x}{c+a\,x}\right|\right)$$

$$\int \mathrm{d}x\,\frac{x^2}{(p\,x+q)\,(c^2-a^2\,x^2)} =$$
$$\frac{1}{c^2\,p^2-a^2\,q^2}\left(\frac{q^2}{p}\ln|p\,x+q|-\frac{c^2\,p}{2\,a^2}\ln\left|c^2-a^2\,x^2\right|+\frac{c\,q}{2\,a}\ln\left|\frac{c-a\,x}{c+a\,x}\right|\right)$$

$$\int \mathrm{d}x\,\sqrt{c^2-a^2\,x^2} = \frac{x}{2}\sqrt{c^2-a^2\,x^2}+\frac{c^2}{2\,a}\arcsin\frac{a\,x}{c}$$

$$\int \frac{\mathrm{d}x}{\sqrt{c^2-a^2\,x^2}} = \frac{1}{a}\arcsin\frac{a\,x}{c}$$

$$\int \mathrm{d}x\,\frac{x}{\sqrt{c^2-a^2\,x^2}} = -\frac{1}{a^2}\sqrt{c^2-a^2\,x^2}$$

$$\int \frac{\mathrm{d}x}{x\sqrt{c^2-a^2\,x^2}} = -\frac{1}{c}\ln\left|\frac{\sqrt{c^2-a^2\,x^2}+c}{x}\right|$$

$$\int \mathrm{d}x\;x\sqrt{c^2-a^2\,x^2} = -\frac{1}{3\,a^2}(c^2-a^2\,x^2)^{3/2}$$

$$\int \mathrm{d}x\;x^2\sqrt{c^2-a^2\,x^2} = -\frac{x}{4\,a^2}(c^2-a^2\,x^2)^{3/2}+\frac{c^2\,x}{8\,a^2}\sqrt{c^2-a^2\,x^2}+\frac{c^4}{8\,a^3}\arcsin\frac{a\,x}{c}$$

$$\int \mathrm{d}x\,\frac{\sqrt{c^2-a^2\,x^2}}{x} = \sqrt{c^2-a^2\,x^2}-c\ln\left|\frac{\sqrt{c^2-a^2\,x^2}+c}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x^2\sqrt{c^2-a^2\,x^2}} = -\frac{\sqrt{c^2-a^2\,x^2}}{c^2\,x}$$

$$\int \mathrm{d}x\;x^n\sqrt{c^2-a^2\,x^2} = -\frac{x^{n-1}\,(c^2-a^2\,x^2)^{3/2}}{(n+2)\,a^2}+\frac{(n-1)\,c^2}{(n+2)\,a^2}\int\mathrm{d}x\;x^{n-2}\sqrt{c^2-a^2\,x^2}\qquad\text{per }\ n>0$$

$$\int \mathrm{d}x\,\frac{\sqrt{c^2-a^2\,x^2}}{x^n} = -\frac{(c^2-a^2\,x^2)^{3/2}}{(n-1)\,c^2\,x^{n-1}}+\frac{(n-4)\,a^2}{(n-1)\,c^2}\int\mathrm{d}x\,\frac{\sqrt{c^2-a^2\,x^2}}{x^{n-2}}\qquad\text{per }\ n>1$$

$$\int \frac{\mathrm{d}x}{x^n\sqrt{c^2-a^2\,x^2}} = -\frac{\sqrt{c^2-a^2\,x^2}}{(n-1)\,c^2\,x^{n-1}}+\frac{(n-2)\,a^2}{(n-1)\,c^2}\int\frac{\mathrm{d}x}{x^{n-2}\sqrt{c^2-a^2\,x^2}}\qquad\text{per }\ n>1$$

$$\int \frac{\mathrm{d}x}{(x-b)\sqrt{p\,x^2+q}} = \left\lfloor t:=\frac{1}{x-b}\right\rfloor = -\mathrm{sign}(x-b)\int\frac{dT}{\sqrt{(p\,b^2+q)\,t^2+2\,p\,b\,t+p}}$$

$$\int \frac{\mathrm{d}x}{(x-b)^n\sqrt{p\,x^2+q}} = \left\lfloor t:=\frac{1}{x-b}\right\rfloor = -\mathrm{sign}(x-b)^n\int\mathrm{d}t\,\frac{t^{n-1}}{\sqrt{(p\,b^2+q)\,t^2+2\,p\,b\,t+p}}$$

$$\int \mathrm{d}x\,(c^2-a^2\,x^2)^{3/2} = \frac{x}{4}(c^2-a^2\,x^2)^{3/2}+\frac{3\,c^2\,x}{8}\sqrt{c^2-a^2\,x^2}+\frac{3\,c^4}{8\,a}\arcsin\frac{a\,x}{c}$$

$$\int \mathrm{d}x\;x\,(c^2-a^2\,x^2)^{3/2} = \frac{1}{5\,a^2}(c^2-a^2\,x^2)^{5/2}$$

$$\int \mathrm{d}x\ \ x\,(c^2-a^2\,x^2)^{3/2} \ = \ \frac{1}{5\,a^2}\,(c^2-a^2\,x^2)^{5/2}$$

$$\int \mathrm{d}x\ \ x^2\,(c^2-a^2\,x^2)^{3/2} \ = \ \frac{x^3}{6}\,(c^2-a^2\,x^2)^{3/2}+\frac{c^2}{2}\int \mathrm{d}x\ \ x^2\,\sqrt{c^2-a^2\,x^2}$$

$$\int \mathrm{d}x\ \ x^3\,(c^2-a^2\,x^2)^{3/2} \ = \ \frac{1}{7\,a^4}\,(c^2-a^2\,x^2)^{7/2}-\frac{c^2}{5\,a^4}\,(c^2-a^2\,x^2)^{5/2}$$

$$\int \mathrm{d}x\,\frac{(c^2-a^2\,x^2)^{3/2}}{x} \ = \ \frac{1}{3}\,(c^2-a^2\,x^2)^{3/2}+c^2\,\sqrt{c^2-a^2\,x^2}-c^3\,\ln\left|\frac{c+\sqrt{c^2-a^2\,x^2}}{x}\right|$$

$$\int \frac{\mathrm{d}x}{(c^2-a^2\,x^2)^{3/2}} \ = \ \frac{x}{c^2\,\sqrt{c^2-a^2\,x^2}}$$

$$\int \mathrm{d}x\,\frac{x}{(c^2-a^2\,x^2)^{3/2}} \ = \ \frac{1}{a^2\,\sqrt{c^2-a^2\,x^2}}$$

$$\int \mathrm{d}x\,\frac{x^2}{(c^2-a^2\,x^2)^{3/2}} \ = \ \frac{x}{a^2\,\sqrt{c^2-a^2\,x^2}}-\frac{1}{a^3}\,\arcsin\,\frac{a\,x}{c}$$

$$\int \mathrm{d}x\,\frac{x^3}{(c^2-a^2\,x^2)^{3/2}} \ = \ \frac{c^2}{a^4\,\sqrt{c^2-a^2\,x^2}}+\frac{1}{a^4}\,\sqrt{c^2-a^2\,x^2}$$

$$\int \frac{\mathrm{d}x}{x\,(c^2-a^2\,x^2)^{3/2}} \ = \ \frac{1}{c^2\,\sqrt{c^2-a^2\,x^2}}+\frac{1}{c^3}\,\ln\left|\frac{\sqrt{c^2-a^2\,x^2}-c}{x}\right|$$

$$\int \frac{\mathrm{d}x}{x^2\,(c^2-a^2\,x^2)^{3/2}} \ = \ \frac{1}{c^4}\left(\frac{\sqrt{c^2-a^2\,x^2}}{x}-\frac{a^2\,x}{\sqrt{c^2-a^2\,x^2}}\right)$$

$$\int \frac{\mathrm{d}x}{x^3\,(c^2-a^2\,x^2)^{3/2}} \ = \ -\frac{1}{2\,c^2}\left(\frac{1}{x^2\,\sqrt{c^2-a^2\,x^2}}-\frac{3\,a^2}{c^2\,\sqrt{c^2-a^2\,x^2}}-\frac{3\,a^2}{c^3}\,\ln\left|\frac{\sqrt{c^2-a^2\,x^2}-c}{x}\right|\right)$$

## W35**b.07** **antiderivate di integrandi con** $a\,x^2+b\,x+c$

Poniamo $k := 4ac-b^2$; osserviamo anche che si può scrivere $a\,x^2+b\,x+c \ = \ a\left(x+\frac{b}{2\,a}\right)^2+c-\frac{b^2}{4\,a}$ e che questa espressione, posto $t := x+\frac{b}{2\,a}$, assume la forma $a\,t^2+\overline{b}$ esaminata in ???
.

$$\int \frac{\mathrm{d}x}{a\,x^2+b\,x+c} \ = \ \begin{cases} \frac{1}{\sqrt{-k}}\,\ln\left|\frac{2\,a\,x+b-\sqrt{-k}}{2\,a\,x+b+\sqrt{-k}}\right| & \text{sse } 4\,a\,c<b^2 \\ \frac{2}{\sqrt{k}}\,\arctan\,\frac{2\,a\,x+b}{\sqrt{k}} & \text{sse } 4\,a\,c>b^2 \\ -\frac{2}{2\,a\,x+b} & \text{sse } 4\,a\,c=b^2 \end{cases}$$

$$\int \mathrm{d}x\,\frac{x}{a\,x^2+b\,x+c} \ = \ \frac{1}{2\,a}\,\ln\left|2\,a\,x^2+b\,x+c\right|-\frac{b}{2\,a}\int \frac{\mathrm{d}x}{a\,x^2+b\,x+c}$$

$$\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^2} \ = \ \frac{2\,a\,x+b}{k(a\,x^2+b\,x+c)}+\frac{2\,a}{k}\int \frac{\mathrm{d}x}{a\,x^2+b\,x+c}$$

$$\int \mathrm{d}x\,\frac{x}{(a\,x^2+b\,x+c)^2} \ = \ -\frac{bx+2\,c}{k(a\,x^2+b\,x+c)}-\frac{b}{k}\int \frac{\mathrm{d}x}{a\,x^2+b\,x+c}$$

$$\int \frac{\mathrm{d}x}{x\,(a\,x^2+b\,x+c)} \ = \ \frac{1}{2\,c}\,\ln\left|\frac{x^2}{a\,x^2+b\,x+c}\right|-\frac{b}{2\,c}\int \frac{\mathrm{d}x}{a\,x^2+b\,x+c}$$

$$\int \frac{\mathrm{d}x}{x^2\,(a\,x^2+b\,x+c)} = \frac{b}{2\,c^2}\,\ln\left|\frac{a\,x^2+b\,x+c}{x^2}\right| - \frac{1}{c\,x} + \left(\frac{b^2}{2\,c^2}-\frac{a}{c}\right)\int \frac{\mathrm{d}x}{a\,x^2+b\,x+c}$$

$$\int \frac{\mathrm{d}x}{\sqrt{a\,x^2+b\,x+c}} = \begin{cases} \frac{1}{\sqrt{a}}\,\ln\left|2\,a\,x+b+2\,\sqrt{a\,x^2+b\,x+c}\right| & \text{sse } a>0 \\ \frac{1}{\sqrt{-a}}\,\arcsin\frac{-2\,a\,x-b}{\sqrt{-k}} & \text{sse } a<0 \end{cases}$$

$$\int \mathrm{d}x\,\frac{x}{\sqrt{a\,x^2+b\,x+c}} = \frac{\sqrt{a\,x^2+b\,x+c}}{a} - \frac{b}{2\,a}\int \frac{\mathrm{d}x}{\sqrt{a\,x^2+b\,x+c}}$$

$$\int \mathrm{d}x\,\frac{x^2}{\sqrt{a\,x^2+b\,x+c}} = \frac{x\,\sqrt{a\,x^2+b\,x+c}}{2\,a} - \frac{3\,b}{4\,a}\int \mathrm{d}x\,\frac{x}{\sqrt{a\,x^2+b\,x+c}} - \frac{c}{2\,a}\int \frac{\mathrm{d}x}{\sqrt{a\,x^2+b\,x+c}}$$

$$\int \frac{\mathrm{d}x}{x\,\sqrt{a\,x^2+b\,x+c}} = \begin{cases} -\frac{1}{\sqrt{c}}\,\ln\left|\frac{\sqrt{a\,x^2+b\,x+c}+\sqrt{c}}{x}+\frac{b}{2\,\sqrt{c}}\right| & \text{sse } c>0 \\ \frac{1}{\sqrt{-c}}\,\arcsin\frac{b\,x+2\,c}{x\,\sqrt{-k}} & \text{sse } c<0 \\ -\frac{2\,\sqrt{a\,x^2+b\,x+c}}{b\,x} & \text{sse } c=0 \end{cases}$$

$$\int \frac{\mathrm{d}x}{x^2\,\sqrt{a\,x^2+b\,x+c}} = -\frac{\sqrt{a\,x^2+b\,x+c}}{c\,x} - \frac{b}{2\,c}\int \frac{\mathrm{d}x}{\sqrt{a\,x^2+b\,x+c}}$$

$$\int \frac{\mathrm{d}x}{(x-d)^n\,\sqrt{a\,x^2+b\,x+c}} = \left\lfloor t := \frac{1}{x-d}\right\rfloor$$

$$= -\mathrm{sign}(x-d)\int \mathrm{d}t\,\frac{t^{n-1}}{\sqrt{(a\,d^2+b\,d+c)^2+(2\,a\,d+b)\,t+a}}$$

$$\int \mathrm{d}x\,\sqrt{a\,x^2+b\,x+c} = \frac{2\,a\,x}{4\,a}\,\sqrt{a\,x^2+b\,x+c} + \frac{k}{8\,a}\int \frac{\mathrm{d}x}{\sqrt{a\,x^2+b\,x+c}}$$

$$\int \mathrm{d}x\;x\,\sqrt{a\,x^2+b\,x+c} = \frac{(a\,x^2+b\,x+c)^{3/2}}{3\,a} - \frac{b}{2\,a}\int \mathrm{d}x\,\sqrt{a\,x^2+b\,x+c}$$

$$\int \mathrm{d}x\;x^2\,\sqrt{a\,x^2+b\,x+c} = \left(x-\frac{5\,b}{6\,a}\right)\frac{(a\,x^2+b\,x+c)^{3/2}}{4\,a} + \frac{5\,b^2-4\,a\,c}{16\,a^2}\int \mathrm{d}x\,\sqrt{a\,x^2+b\,x+c}$$

$$\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^{3/2}} = \frac{2(2\,a\,x+b)}{k\,\sqrt{a\,x^2+b\,x+c}}$$

$$\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^{n+1}} = \frac{2\,a\,x+b)}{k\,n\,(a\,x^2+b\,x+c)^n} + \frac{2(2\,n-1)\,a}{k\,n}\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^n}$$

$$\int \mathrm{d}x\,\frac{x}{(a\,x^2+b\,x+c)^{n+1}} = -\frac{b\,x+2\,c}{k\,n\,(a\,x^2+b\,x+c)^n} - \frac{2(2\,n-1)\,b}{k\,n}\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^n}$$

$$\int \mathrm{d}x\,\frac{x^m}{(a\,x^2+b\,x+c)^n} = \begin{cases} -\frac{x^{m-1}}{a\,(2\,n-m-1)\,(a\,x^2+b\,x+c)^{n-1}} - \frac{n-m)\,b}{(2\,n-m-1)\,a}\int \mathrm{d}x\,\frac{x^{m-1}}{(a\,x^2+b\,x+c)^n} \\ \qquad + \frac{m-1)\,c}{(2\,n-m-1)\,a}\int \mathrm{d}x\,\frac{x^{m-2}}{(a\,x^2+b\,x+c)^n} \quad \text{sse } m \neq 2\,n-1 \\ \frac{1}{a}\int \mathrm{d}x\,\frac{x^{m-2}}{(a\,x^2+b\,x+c)^{n-1}} - \frac{b}{a}\int \mathrm{d}x\,\frac{x^{m-1}}{(a\,x^2+b\,x+c)^n} - \frac{c}{a}\int \mathrm{d}x\,\frac{x^{m-2}}{(a\,x^2+b\,x+c)^n} \end{cases}$$

$$\int \frac{\mathrm{d}x}{x^m\,(a\,x^2+b\,x+c)^n} = -\frac{1}{(m-1)\,c\,x^{m-1}\,(a\,x^2+b\,x+c)^{n-1}}$$

$$-\frac{(n+m-2)\,b}{(m-1)\,c}\int \frac{\mathrm{d}x}{x^{m-1}\,(a\,x^2+b\,x+c)^n} - \frac{(2\,n+m-3)\,a}{(m-1)\,c}\int \frac{\mathrm{d}x}{x^{m-2}\,(a\,x^2+b\,x+c)^n}$$

$$\int \frac{\mathrm{d}x}{x\,(a\,x^2+b\,x+c)^n} = \frac{1}{2\,c\,(n-1)(a\,x^2+b\,x+c)^{n-1}} - \frac{b}{2\,c}\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^n}$$

$$+\frac{1}{c}\int \frac{\mathrm{d}x}{(a\,x^2+b\,x+c)^{n-1}}$$

$$\int \mathrm{d}x\,(a\,x^2+b\,x+c)^n \;=\; \frac{(2\,a\,x+b)\,(a\,x^2+b\,x+c)^n}{2\,(2\,n+1)\,a} - \frac{n\,k}{2(2\,n+1)\,a}\int \mathrm{d}x\,(a\,x^2+b\,x+c)^{n-1}$$

$$\int \mathrm{d}x\;\; x\,(a\,x^2+b\,x+c)^n \;=\; \frac{(a\,x^2+b\,x+c)^{n+1}}{2\,(n+1)\,a} - \frac{b}{2\,a}\int \mathrm{d}x\,(a\,x^2+b\,x+c)^n$$

## W35**b.08** antiderivate di integrandi con $x^3 \pm a^3$

Presentiamo solo formule riguardanti $x^3+a^3$, ma osserviamo che le corrispondenti formule riguardanti $x^3-a^3$ si ottengono dalle seguenti cambiando $a$ in $-a$.

$$\int \frac{\mathrm{d}x}{x^3+a^3} \;=\; \frac{1}{3\,a^2}\left(\frac{1}{2}\,\ln\frac{(x+a)^2}{x^2-a\,x+a^2}+\sqrt{3}\,\arctan\frac{2\,x-a}{a\,\sqrt{3}}\right) \;=:\; I_1$$

$$\int \frac{\mathrm{d}x}{(x^3+a^3)^2} \;=\; \frac{x}{a^3(x^3+a^3)}+\frac{2}{3\,a^3}\int \frac{\mathrm{d}x}{x^3+a^3} \;=\; ...\; I_1$$

$$\int \frac{\mathrm{d}x}{x^3+a^3} \;=\; \frac{1}{3\,a}\left(\frac{1}{2}\,\ln\frac{(x+a)^2}{x^2-a\,x+a^2}+\sqrt{3}\,\arctan\frac{2\,x-a}{a\,\sqrt{3}}\right) \;=:\; I_3$$

$$\int \mathrm{d}x\;\frac{x^2}{x^3+a^3} \;=\; \frac{1}{3}\,\ln|x^3+a^3|$$

$$\int \frac{\mathrm{d}x}{x(x^3+a^3)} \;=\; \frac{1}{3\,a^3}\;\ln\left|\frac{x^3}{x^3+a^3}\right|$$

$$\int \frac{\mathrm{d}x}{x^2(x^3+a^3)} \;=\; \frac{1}{a^3\,x}-\frac{1}{a^3}\int \mathrm{d}x\,\frac{x}{x^3+a^3} \;=\; ...\; I_3$$

$$\int \frac{\mathrm{d}x}{x^2(x^3+a^3)} \;=\; \frac{1}{a^3\,x}-\frac{1}{a^3}\int \mathrm{d}x\,\frac{x}{x^3+a^3} \;=\; ...\; I_3$$

## W35**b.09** antiderivate di integrandi con $x^4 \pm a^4$

$$\int \frac{\mathrm{d}x}{x^4+a^4} \;=\; \frac{1}{2\,\sqrt{2}\,a^3}\left(\frac{1}{2}\,\ln\frac{x^2+\sqrt{2}\,a\,x+a^2}{x^2-\sqrt{2}\,a\,x+a^2}+\arctan\frac{\sqrt{2}\,a\,x}{a^2-x^2}+\pi\right)$$

$$\int \frac{\mathrm{d}x}{x^4-a^4} \;=\; \frac{1}{2\,a^3}\left(\frac{1}{2}\,\ln\left|\frac{x-a}{x+a}\right|-\arctan\frac{x}{a}\right)$$

$$\int \mathrm{d}x\;\frac{x}{x^4+a^4} \;=\; \frac{1}{2\,a^2}\;\arctan\frac{x^2}{a^2}$$

$$\int \mathrm{d}x\;\frac{x}{x^4-a^4} \;=\; \frac{1}{4\,a^2}\;\ln\left|\frac{x^2-a^2}{x^2+a^2}\right|$$

$$\int \mathrm{d}x\;\frac{x^2}{x^4+a^4} \;=\; \frac{1}{2\,\sqrt{2}a}\left(\frac{1}{2}\,\ln\frac{x^2-\sqrt{2}ax+a^2}{x^2+\sqrt{2}ax+a^2}+\arctan\frac{\sqrt{2}\,a\,x}{a^2-x^2}+\pi\right)$$

$$\int \mathrm{d}x\;\frac{x^2}{x^4-a^4} \;=\; \frac{1}{2a}\left(\frac{1}{2}\,\ln\left|\frac{x-a}{x+a}\right|+\arctan\frac{x}{a}\right)$$

$$\int \mathrm{d}x\;\frac{x^3}{x^4\pm a^4} \;=\; \frac{1}{4}\,\ln\left|x^4\pm a^4\right|$$

## W35**b.10** antiderivate di integrandi con $a\,x^n + b$

$$\int \frac{\mathrm{d}x}{x(a\,x^n+b)} \;=\; \frac{1}{b\,n}\,\ln\left|\frac{x^n}{u}\right|$$

$$\int \frac{\mathrm{d}x}{x\,\sqrt{a\,x^n+b}} \;=\; \begin{cases} \frac{1}{n\,\sqrt{b}}\,\ln\left|\frac{\sqrt{u}-\sqrt{b}}{\sqrt{u}+\sqrt{b}}\right| & \text{sse } b>0 \\ \frac{2}{n\,\sqrt{-b}}\,\arctan\sqrt{\frac{u}{-b}} & \text{sse } b<0 \end{cases}$$

Introduciamo $u := a\,x^n + b$ e consideriamo i parametri $m, n, p \in \mathbb{R}$ .

$$\int \mathrm{d}x\;\, x^m(a\,x^n+b)^p \;=\; \frac{1}{m+np+1}\left(x^{m+1}\,u^p + n\,p\,b\int \mathrm{d}x\;\, x^m\,u^{p-1}\right)$$

$$=\; \frac{1}{b\,n(p+1)}\left(-x^{m+1}\,u^{p+1} + (m+n\,p+n+1)\int \mathrm{d}x\;\, x^m\,u^{p+1}\right)$$

$$=\; \frac{1}{a(m+n\,p+1)}\left(x^{m-n+1}\,u^{p+1} - (m-n+1)\,b\int \mathrm{d}x\;\, x^{m-n}\,u^p\right)$$

$$=\; \frac{1}{b(m+1)}\left(x^{m+1}\,u^{p+1} - (m-n\,p+n+1)\,b\int \mathrm{d}x\;\, x^{m+n}\,u^p\right)$$

## W35 c. antiderivate di integrandi trascendenti

Le formule che seguono possono essere utilizzate anche per integrandi contenenti richiami a tangenti, cotangenti, secanti e cosecanti, pur di tenere conto che

$$\tan x = \frac{\sin x}{\cos x} \quad , \quad \cot x = \frac{\cos x}{\sin x} \quad , \quad \sec x = \frac{1}{\cos x} \quad , \quad \csc x = \frac{1}{\sin x}$$

Nelle espressioni delle antiderivate le costanti additive arbitrarie come `C` e $\mathtt{C}_1$ sono indicate solo in presenza di risultati formalmente diversi.

### W35c.01 antiderivate di integrandi con seno e/o coseno

$$\int dx \ \sin x = -\cos x \quad , \quad \int dx \ \sin a\,x = -\frac{1}{a}\cos a\,x \quad ,$$

$$\int dx \ \cos x = \sin x \quad , \quad \int dx \ \cos a\,x = \frac{1}{a}\sin a\,x$$

$$\int dx \ \sin^2 a\,x = \frac{x}{2} - \frac{\sin 2\,a\,x}{4\,a} \quad , \quad \int dx \ \cos^2 a\,x = \frac{x}{2} + \frac{\sin 2\,a\,x}{4\,a}$$

$$\int dx \ \sin^3 a\,x = -\frac{1}{a}\cos a\,x + \frac{1}{3\,a}\cos 3\,a\,x \quad , \quad \int dx \ \cos^3 a\,x = \frac{1}{a}\sin a\,x - \frac{1}{3\,a}\sin 3\,a\,x$$

$$\int dx \ \sin^4 a\,x = \frac{3\,x}{8} - \frac{\sin 2\,a\,x}{4\,a} + \frac{\sin 4\,a\,x}{32\,a} \quad , \quad \int dx \ \cos^4 a\,x = \frac{3\,x}{8} + \frac{\sin 2\,a\,x}{4\,a} + \frac{\sin 4\,a\,x}{32\,a}$$

$$\int dx \ \sin^n a\,x = -\frac{1}{n\,a}\sin^{n-1} a\,x \cos a\,x + \frac{n-1}{n}\int dx \ \sin^{n-2} a\,x$$

$$\int dx \ \cos^n a\,x = \frac{1}{n\,a}\sin a\,x \cos^{n-1} a\,x + \frac{n-1}{n}\int dx \ \cos^{n-2} a\,x$$

$$\int dx \ \sin a\,x \cos^n a\,x = \begin{cases} -\frac{\cos^{n+1} a\,x}{a(n+1)} & \text{sse } n \neq -1 \\ -\frac{1}{a}\ln|\cos, a\,x| & \text{sse n=-1} \end{cases}$$

$$\int dx \ \sin^n a\,x \cos a\,x = \begin{cases} -\frac{\sin^{n+1} a\,x}{a(n+1)} & \text{sse } n \neq -1 \\ -\frac{1}{a}\ln|\sin a\,x| & \text{sse n=-1} \end{cases}$$

$$\int dx \ \sin^m x \cos^n x = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int dx \ \sin^m x \cos^{n-2} x$$

$$= -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int dx \ \sin^{m-2} x \cos^n x$$

$$\int dx \ x \sin a\,x = \frac{1}{a^2}\big(\sin a\,x - a\,x \cos a\,x\big)$$

$$\int dx \ x \cos a\,x = \frac{1}{a^2}\big(\cos a\,x + a\,x \sin a\,x\big)$$

$$\int dx \ x^2 \sin a\,x = \frac{1}{a^3}\big(2\cos a\,x + 2\,a\,x \sin a\,x - a^2\,x^2 \cos a\,x\big)$$

$$\int dx \ x^2 \cos a\,x = \frac{1}{a^3}\big(-2\sin a\,x + 2\,a\,x \cos a\,x + a^2\,x^2 \sin a\,x\big)$$

$$\int dx \ x^n \sin a\,x = -\frac{1}{a}x^n \cos a\,x + \frac{n}{a}\int dx \ x^{n-1}\cos a\,x$$

$$\int dx \ x^n \cos a\,x = -\frac{1}{a}x^n \sin a\,x - \frac{n}{a}\int dx \ x^{n-1}\sin a\,x$$

$$\int \mathrm{d}x\ \ x \sin^2\ a\,x \ = \ \frac{x^2}{4} - \frac{x \sin\ 2\,a\,x}{4\,a} - \frac{\cos\ 2\,a\,x}{8\,a^2}$$

$$\int \mathrm{d}x\ \ x \cos^2\ a\,x \ = \ \frac{x^2}{4} + \frac{x \sin\ 2\,a\,x}{4\,a} + \frac{\cos\ 2\,a\,x}{8\,a^2}$$

$$\int \mathrm{d}x\ \ \sin\,m\,x\ \sin\,n\,x \ = \ \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} \quad \text{per} \quad m^2 \neq n^2$$

$$\int \mathrm{d}x\ \ \sin\,m\,x\ \cos\,n\,x \ = \ -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} \quad \text{per} \quad m^2 \neq n^2$$

$$\int \mathrm{d}x\ \ \cos\,m\,x\ \cos\,n\,x \ = \ \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} \quad \text{per} \quad m^2 \neq n^2$$

## W35c.02 antiderivate di integrandi con (co)tangente e (co)secante

$$\int \mathrm{d}x\ \ \tan\,a\,x \ = \ \frac{1}{a}\ \ln|\cos\,a\,x| + \mathtt{C} \ = \ \frac{1}{a}\ \ln|\sec a\,x| + \mathtt{C}_1$$

$$\int \mathrm{d}x\ \ \cot\,a\,x \ = \ \int \frac{\mathrm{d}x}{\tan\,a\,x} \ = \ \frac{1}{a}\ \ln|\sin\,a\,x|x + \mathtt{C} \ = \ \frac{1}{a}\ \ln|\csc\ a\,x| + \mathtt{C}_1$$

$$\int \mathrm{d}x\ \ \tan^2\ a\,x \ = \ \frac{1}{a}\ \tan\,a\,x - x$$

$$\int \mathrm{d}x\ \ \tan^3\ a\,x \ = \ \frac{1}{2\,a}\ \tan^2\ a\,x + \frac{1}{a}\ \ln\ |\cos\,a\,x|$$

$$\int \mathrm{d}x\ \ \tan^n\ a\,x \ = \ \frac{1}{(n-1)\,a}\ \tan^{n-1}\ a\,x - \int \mathrm{d}x\ \ \tan^{n-2}\ a\,x$$

$$\int \mathrm{d}x\ \ \tan^n\ a\,x \sec^2 a\,x \ = \ \int \mathrm{d}x \frac{\tan^n\ a\,x}{\cos^2\ a\,x} \ = \ \frac{1}{(n+1)\,a}\ \tan^{n+1}\ a\,x \quad \text{per} \quad n \neq -1$$

$$\int \mathrm{d}x\ \ \cot^2\ a\,x \ = \ -\frac{1}{a}\ \cot\,a\,x - x$$

$$\int \mathrm{d}x\ \ \cot^3\ a\,x \ = \ \frac{1}{2\,a}\ \cot^2\ a\,x - \frac{1}{a}\ \ln\ |\sin\,a\,x|$$

$$\int \mathrm{d}x\ \ \cot^n\ a\,x \ = \ -\frac{1}{(n-1)\,a}\ \cot^{n-1}\ a\,x - \int \mathrm{d}x\ \ \cot^{n-2}\ a\,x$$

$$\int \mathrm{d}x\ \ \cot^n\ a\,x \csc^{\,2} a\,x \ = \ \int \mathrm{d}x \frac{\cot^n\ a\,x}{\sin^2\ a\,x} \ = \ \frac{1}{(n+1)\,a}\ \cot^{n+1}\ a\,x \quad \text{per} \quad n \neq -1$$

$$\int \mathrm{d}x \frac{\sec^2 a\,x}{\tan\,a\,x} \ = \ \int \frac{\mathrm{d}x}{\cos^2\ a\,x\ \tan\,a\,x} \ = \ \frac{1}{a}\ \ln|\tan\,a\,x|$$

$$\int \mathrm{d}x \frac{\csc^{\,2} a\,x}{\cot\,a\,x} \ = \ \int \frac{\mathrm{d}x}{\sin^2\ a\,x\ \cot\,a\,x} \ = \ -\frac{1}{a}\ \ln|\cot\,a\,x|$$

$$\int \mathrm{d}x\ \ x \tan^2\ a\,x \ = \ \frac{x}{a}\ \tan\,a\,x + \frac{1}{a^2}\ \ln|\cos\,a\,x| - \frac{x^2}{2}$$

$$\int \mathrm{d}x\ \ x \cot^2\ a\,x \ = \ -\frac{x}{a}\ \cot\,a\,x + \frac{1}{a^2}\ \ln|\sin\,a\,x| - \frac{x^2}{2}$$

$$\int \frac{\mathrm{d}x}{b + c\ \tan\,x} \ = \ \frac{1}{b^2+c^2}\left(b\,x + c\ \ln|b\ \cos\,x + c\ \sin\,x|\right)$$

$$\int \frac{\mathrm{d}x}{\sqrt{b + c\ \tan^2\,x}} \ = \ \frac{1}{\sqrt{b-c}}\ \arcsin\left(\sqrt{\frac{b-c}{b}}\ \sin\,x\right) \quad \text{per} \quad b>0\ ,\ b^2 > c^2$$

$$\int \frac{\mathrm{d}x}{\sin^2 a\,x} = \int \mathrm{d}x\ \csc^2 a\,x = -\frac{1}{a}\cot a\,x$$

$$\int \frac{\mathrm{d}x}{\cos^2 a\,x} = \int \mathrm{d}x\ \sec^2 a\,x = \frac{1}{a}\tan a\,x$$

$$\int \mathrm{d}x\ \sec a\,x = \int \frac{\mathrm{d}x}{\cos a\,x} = \frac{1}{a}\ln\left|\tan\left(\frac{a\,x}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \mathrm{d}x\ \csc a\,x = \int \frac{\mathrm{d}x}{\sin a\,x} = \frac{1}{a}\ln\left|\tan\frac{a\,x}{2}\right|$$

$$\int \mathrm{d}x\ \sec x = \ln|\sec x + \tan x| \quad , \quad \int \mathrm{d}x\ \csc x = \ln|\csc x| - \cot x|$$

$$\int \mathrm{d}x\ \sec a\,x \tan a\,x = \int \mathrm{d}x\,\frac{\sin a\,x}{\cos^2 a\,x} = \frac{1}{a}\sec a\,x = \frac{1}{a\cos a\,x}$$

$$\int \mathrm{d}x\ \csc a\,x\cot a\,x = \int \mathrm{d}x\,\frac{\cos a\,x}{\sin^2 a\,x} = -\frac{1}{a}\csc x = -\frac{1}{a\sin a\,x}$$

$$\int \frac{\mathrm{d}x}{\sin^n a\,x} = -\frac{\cos a\,x}{a\,(n-1)\sin^{n-1} a\,x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\sin^{n-2} a\,x}$$

$$\int \frac{\mathrm{d}x}{\cos^n a\,x} = -\frac{\sin a\,x}{a\,(n-1)\cos^{n-1} a\,x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\cos^{n-2} a\,x}$$

$$\int \frac{\mathrm{d}x}{\sin a\,x\cos a\,x} = \frac{1}{a}\ln|\tan a\,x|$$

$$\int \frac{\mathrm{d}x}{\sin a\,x\cos^2 a\,x} = \frac{1}{a}\left(\frac{1}{\cos a\,x} + \ln\left|\tan\frac{a\,x}{2}\right|\right)$$

$$\int \frac{\mathrm{d}x}{\sin^2 a\,x\cos a\,x} = \frac{1}{a}\left(-\frac{1}{\sin a\,x} + \ln\left|\tan\left(\frac{a\,x}{2}+\frac{\pi}{4}\right)\right|\right)$$

$$\begin{aligned}\int \frac{\mathrm{d}x}{\sin^m x\cos^n x} &= -\frac{1}{(m-1)\sin^{m-1} x\cos^{m-1} x} + \frac{m+n-2}{m-1}\int \frac{\mathrm{d}x}{\sin^{m-2} x\cos^n x}\\ &= -\frac{1}{(m-1)\sin^{m-1} x\cos^{m-1} x} - \frac{m+n-2}{n-1}\int \frac{\mathrm{d}x}{\sin^m x\cos^{n-2} x}\end{aligned}$$

$$\begin{aligned}\int \mathrm{d}x\,\frac{\sin^m x}{\cos^n x} &= \frac{\sin^{m+1} x}{(n-1)\cos^{n-1} x} + \frac{m+n-2}{n-1}\int \mathrm{d}x\,\frac{\sin^m x}{\cos^{n-2} x}\\ &= -\frac{\sin^{m-1} x}{(m-n)\cos^{n-1} x} + \frac{m-1}{m-n}\int \mathrm{d}x\,\frac{\sin^{m-2} x}{\cos^n x}\end{aligned}$$

$$\begin{aligned}\int \mathrm{d}x\,\frac{\cos^m x}{\sin^n x} &= -\frac{\cos^{n+1} x}{(m-1)\sin^{m-1} x} - \frac{m+n-2}{m-1}\int \mathrm{d}x\,\frac{\cos^n x}{\sin^{m-2} x}\\ &= \frac{\cos^{m-1} x}{(m-n)\sin^{m-1} x} + \frac{n-1}{n-m}\int \mathrm{d}x\,\frac{\cos^{n-2} x}{\sin^n x}\end{aligned}$$

$$\int \mathrm{d}x\,\frac{x}{\sin^2 a\,x} = -\frac{x}{a}\cot a\,x + \frac{1}{a^2}\ln|\sin a\,x|$$

$$\int \mathrm{d}x\,\frac{x}{\cos^2 a\,x} = \frac{x}{a}\tan a\,x + \frac{1}{a^2}\ln|\cos a\,x|$$

$$\int \frac{\mathrm{d}x}{1+\sin a\,x} = -\frac{1}{a}\tan\left(\frac{\pi}{4}-\frac{a\,x}{2}\right)$$

$$\int \frac{\mathrm{d}x}{1-\sin a\,x} = \frac{1}{a}\tan\left(\frac{\pi}{4}+\frac{a\,x}{2}\right)$$

$$\int \frac{\mathrm{d}x}{1+\cos\,a\,x} \;=\; \frac{1}{a}\,\tan\,\frac{a\,x}{2}$$

$$\int \frac{\mathrm{d}x}{1-\cos\,a\,x} \;=\; -\frac{1}{a}\,\cot\,\frac{a\,x}{2}$$

$$\int \frac{\mathrm{d}x}{b+c\,\sin\,a\,x} \;=\; \begin{cases} \frac{2}{a\,\sqrt{b^2-c^2}}\;\arctan\,\frac{b\,\tan(a\,x/2)+c}{\sqrt{b^2-c^2}} & \text{sse}\quad b^2>c^2 \\ \frac{1}{a\,\sqrt{c^2-b^2}}\;\ln\left|\frac{b\,\tan(a\,x/2)+c-\sqrt{c^2-b^2}}{b\,\tan(a\,x/2)+c+\sqrt{c^2-b^2}}\right| & \text{sse}\quad b^2<c^2 \end{cases}$$

$$\int \frac{\mathrm{d}x}{\sin\,x\,(b+c\,\sin\,x)} \;=\; \frac{1}{b}\,\ln\left|\tan\,\frac{x}{2}\right| - \frac{c}{b}\int \frac{\mathrm{d}x}{b+c\,\sin\,a\,x}$$

$$\int \frac{\mathrm{d}x}{\sin\,x\,(1+\sin\,x)} \;=\; \ln\left|\tan\,\frac{x}{2}\right| - \tan\left(\frac{x}{2}-\frac{\pi}{4}\right)$$

$$\int \frac{\mathrm{d}x}{\sin\,x\,(1-\sin\,x)} \;=\; \ln\left|\tan\,\frac{x}{2}\right| + \tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\int \frac{\mathrm{d}x}{\sin\,x\,(1-\sin\,x)} \;=\; \ln\left|\tan\,\frac{x}{2}\right| + \tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\int \frac{\mathrm{d}x}{(b+c\,\sin\,x)^2} \;=\; \frac{c\,\cos\,x}{(b^2-c^2)\,(b+c\,\sin\,x)} + \frac{b}{b^2-c^2}\int \frac{\mathrm{d}x}{b+c\,\sin\,x}$$

$$\int \mathrm{d}x\,\frac{\sin\,x}{(b+c\,\sin\,x)^2} \;=\; \frac{b\,\cos\,x}{(b^2-c^2)\,(b+c\,\sin\,x)} + \frac{c}{c^2-b^2}\int \frac{\mathrm{d}x}{b+c\,\sin\,x}$$

$$\int \mathrm{d}x\,\frac{\cos\,x}{(b+c\,\sin\,x)^2} \;=\; -\frac{1}{c\,(b+c\,\sin\,x)}$$

$$\int \frac{\mathrm{d}x}{b+c\,\cos\,a\,x} \;=\; \begin{cases} \frac{2}{a\,\sqrt{b^2-c^2}}\;\arctan\,\frac{(b-c)\,\tan(a\,x/2)}{\sqrt{b^2-c^2}} & \text{sse}\quad b^2>c^2 \\ \frac{1}{a\,\sqrt{c^2-b^2}}\;\ln\left|\frac{(c-b)\,\tan(a\,x/2)+\sqrt{c^2-b^2}}{(c-b)\,\tan(a\,x/2)-\sqrt{c^2-b^2}}\right| & \text{sse}\quad b^2<c^2 \end{cases}$$

$$\int \frac{\mathrm{d}x}{\cos\,x\,(b+c\,\cos\,x)} \;=\; \frac{1}{b}\,\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| - \frac{c}{b}\int \frac{\mathrm{d}x}{b+c\,\sin\,a\,x}$$

$$\int \frac{\mathrm{d}x}{\cos\,x\,(1+\cos\,x)} \;=\; \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| - \tan\,\frac{x}{2}$$

$$\int \frac{\mathrm{d}x}{\cos\,x\,(1-\cos\,x)} \;=\; \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| - \cot\,\frac{x}{2}$$

$$\int \frac{\mathrm{d}x}{(b+c\,\cos\,x)^2} \;=\; \frac{c\,\sin\,x}{(c^2-b^2)\,(b+c\,\cos\,x)} - \frac{b}{c^2-b^2}\int \frac{\mathrm{d}x}{b+c\,\cos\,x}$$

$$\int \mathrm{d}x\,\frac{\cos\,x}{(b+c\,\cos\,x)^2} \;=\; \frac{b\,\sin\,x}{(b^2-c^2)\,(b+c\,\cos\,x)} - \frac{c}{b^2-c^2}\int \frac{\mathrm{d}x}{b+c\,\sin\,x}$$

$$\int \mathrm{d}x\,\frac{\sin\,x}{(b+c\,\cos\,x)^2} \;=\; \frac{1}{c\,(b+c\,\cos\,x)}$$

Dati $b$ e $c$ reali nonnulli, introduciamo $r := \sqrt{b^2+c^2}$ e $\phi \;:=\; \arctan\,\frac{b}{c}$

$$\int \frac{\mathrm{d}x}{b\,\cos\,x+c\,\sin\,x} \;=\; \frac{1}{r}\,\ln\left|\tan\,\frac{x+\phi}{2}\right| \qquad \text{per}\quad c>0$$

$$\int \frac{\mathrm{d}x}{a+b\,\cos\,x+c\,\sin\,x} \;=\; \Big\lfloor\, t := x+\phi \,\Big\rfloor \;=\; \int \frac{dT}{a+r\,\sin\,t}$$

$$\int \mathrm{d}x\,\frac{\sin\,a\,x}{b+c\,\cos\,x} \;=\; -\frac{1}{a\,c}\,\ln\,|b+c\,\cos\,a\,x|$$

$$\int \mathrm{d}x\,\frac{\cos\,a\,x}{b+c\,\sin\,x} \;=\; -\frac{1}{a\,c}\,\ln\,|b+c\,\sin\,a\,x|$$

$$\int \frac{\mathrm{d}x}{a\ \sin^2\ a+b} \ = \ \int \frac{\mathrm{d}x}{(a+b)\ \sin^2\ x + b\ \cos^2\ x}$$

$$\int \frac{\mathrm{d}x}{a\ \cos^2\ a+b} \ = \ \int \frac{\mathrm{d}x}{(a+b)\ \cos^2\ x + b\ \sin^2\ x}$$

$$\int \frac{\mathrm{d}x}{a^2\ \cos^2\ x + b^2\ \sin^2} \ = \ \frac{1}{a\,b}\ \arctan\left(\frac{b}{a}\ \tan\ x\right)$$

$$\int \frac{\mathrm{d}x}{a^2\ \cos^2\ x - b^2\ \sin^2} \ = \ \frac{1}{2\,a\,b}\ \ln\left|\frac{b\ \tan\ x + a}{b\ \tan\ x - a}\right|$$

$$\int \mathrm{d}x\,\frac{\sin\ x}{a\ \cos^2\ x+b} \ = \ \Big\lfloor\ t := \cos\ x\ \Big\rfloor \ = \ -\int \frac{dT}{a\,t^2+b}$$

$$\int \mathrm{d}x\,\frac{\cos\ x}{a\ \sin^2\ x+b} \ = \ \Big\lfloor\ t := \sin\ x\ \Big\rfloor \ = \ \int \frac{dT}{a\,t^2+b}$$

Da alcune delle presenti formule che riguardano $\sin^2\ x$ si possono ricavare corrispondenti formule utilizzando la $\cos^2\ x \ = \ 1-\sin^2\ x$ .

Consideriamo il parametro $a \in \mathbb{R}_+$

$$\int \mathrm{d}x\ \ \sin\ x\,\sqrt{a\ \sin^2\ x+b} \ = \ -\frac{\cos\ x}{2}\ \sqrt{a\ \sin^2\ x+b}+b-\frac{a+b}{2\,\sqrt{a}}\ \arcsin\ \frac{\sqrt{a}\ \cos\ x}{\sqrt{a+b}}$$

$$\int \mathrm{d}x\ \ \sin\ x\,\sqrt{b-a\ \sin^2\ x} \ = \ -\frac{\cos\ x}{2}\ \sqrt{b-a\ \sin^2\ x}-\frac{a-b}{2\,\sqrt{a}}\ \ln\left|\sqrt{a}\ \cos\ x+\sqrt{b-a\ \sin^2\ x}\right|$$

$$\int \mathrm{d}x\,\frac{\sin\ x}{\sqrt{a\ \sin^2\ x+b}} \ = \ -\frac{1}{\sqrt{a}}\ \arcsin\ \frac{\sqrt{a}\ \cos\ x}{\sqrt{a+b}}$$

$$\int \mathrm{d}x\,\frac{\sin\ x}{\sqrt{b-a\ \sin^2\ x}} \ = \ -\frac{1}{\sqrt{a}}\ \ln\left|\sqrt{a}\ \cos\ x+\sqrt{b-a\ \sin^2\ x}\right|$$

$$\int \mathrm{d}x\ \ \cos\ x\,\sqrt{a\ \sin^2\ x+b} \ = \ \frac{\sin\ x}{2}\ \sqrt{a\ \sin^2\ x+b}+\frac{b}{2\,\sqrt{a}}\ \ln\left|\sqrt{a}\ \sin\ x+\sqrt{a\ \sin^2\ x+b}\right|$$

$$\int \mathrm{d}x\ \ \cos\ x\,\sqrt{b-a\ \sin^2\ x} \ = \ \frac{\sin\ x}{2}\ \sqrt{b-a\ \sin^2\ x}+\frac{b}{2\,\sqrt{a}}\ \arcsin\left(\sqrt{\frac{a}{b}}\ \sin\ x\right)$$

$$\int \mathrm{d}x\,\frac{\cos\ x}{\sqrt{a\ \sin^2\ x+b}} \ = \ \frac{1}{\sqrt{a}}\ \ln\left|\sqrt{a}\ \sin\ x+\sqrt{a\ \sin^2\ x+b}\right|$$

$$\int \mathrm{d}x\,\frac{\cos\ x}{\sqrt{b-a\ \sin^2\ x}} \ = \ \frac{1}{\sqrt{a}}\ \arcsin\left(\sqrt{\frac{a}{b}}\ \sin\ x\right)$$

## W35c.03 **antiderivate di integrandi con trigonometriche inverse**

Ricordiamo che: $\displaystyle\int \mathrm{d}x\,\frac{1}{1+x^2} \ = \ \arctan\ x$ , $\displaystyle\int \mathrm{d}x\,\frac{1}{\sqrt{1-x^2}} \ = \ \arcsin\ x$

$$\int \mathrm{d}x\ \ \arcsin\ a\,x \ = \ x\ \arcsin\ a\,x+\frac{1}{a}\ \sqrt{1-a^2\,x^2}$$

$$\int \mathrm{d}x\,(\arcsin\ a\,x)^2 \ = \ x\,(\arcsin\ a\,x)^2-2x+\frac{2}{a}\ \sqrt{1-a^2\,x^2}\ \arcsin\ a\,x$$

$$\int \mathrm{d}x\ \ x\ \arcsin\ a\,x \ = \ \frac{1}{4\,a^2}\ \left(2\,a^2\,x^2\ \arcsin\ a\,x-\arcsin\ a\,x+a\,x\,\sqrt{1-a^2\,x^2}\right)$$

$$\int dx\ x^2 \arcsin a\,x \;=\; \frac{1}{9\,a^3}\,\left(3\,a^3\,x^3 \arcsin a\,x + (a^2\,x^2+2)\,\sqrt{1-a^2\,x^2}\right)$$

$$\int dx\,\frac{\arcsin a\,x}{x^2} \;=\; -\frac{1}{x}\arcsin a\,x - a\,\ln\left|\frac{1+\sqrt{1-a^2\,x^2}}{a\,x}\right|$$

$$\int dx\ \arccos a\,x \;=\; x\arccos a\,x - \frac{1}{a}\sqrt{1-a^2\,x^2}$$

$$\int dx\,(\arccos a\,x)^2 \;=\; x\,(\arccos a\,x)^2 - 2x - \frac{2}{a}\sqrt{1-a^2\,x^2}\arccos a\,x$$

$$\int dx\ x\arccos a\,x \;=\; \frac{1}{4\,a^2}\left(2\,a^2\,x^2\arccos a\,x - \arccos a\,x - a\,x\,\sqrt{1-a^2\,x^2}\right)$$

$$\int dx\ x^2\arccos a\,x \;=\; \frac{1}{9\,a^3}\left(3\,a^3\,x^3\arccos a\,x - (a^2\,x^2+2)\,\sqrt{1-a^2\,x^2}\right)$$

$$\int dx\,\frac{\arccos a\,x}{x^2} \;=\; -\frac{1}{x}\arccos a\,x + a\,\ln\left|\frac{1+\sqrt{1-a^2\,x^2}}{a\,x}\right|$$

$$\int dx\ \arctan a\,x \;=\; \frac{1}{2\,a}\,\left(2\,a\,x\,\arctan a\,x - \ln(1+a^2\,x^2)\right)$$

$$\int dx\ \operatorname{arccot} a\,x \;=\; \frac{1}{2\,a}\,\left(2\,a\,x\operatorname{arccot} a\,x + \ln(1+a^2\,x^2)\right)$$

$$\int dx\ x\,\arctan a\,x \;=\; \frac{1}{2\,a^2}\,\left((1+a^2\,x^2)\,\arctan a\,x - a\,x\right)$$

$$\int dx\ x^2\,\arctan a\,x \;=\; \frac{1}{6\,a^3}\,\left(2\,a^3\,x^3\,\arctan a\,x - a^2\,x^2 + \ln(1+a^2\,x^2)\right)$$

$$\int dx\,\frac{\arctan a\,x}{x^2} \;=\; -\frac{1}{x}\,\arctan a\,x - \frac{a}{2}\,\ln\frac{1+a^2\,x^2}{a^2\,x^2}$$

$$\int dx\ \operatorname{arcsec} a\,x \;=\; x\operatorname{arcsec} a\,x - \frac{1}{a}\,\ln\left|a\,x+\sqrt{a^2\,x^2-1}\right|$$

$$\int dx\ \operatorname{arccsc} a\,x \;=\; x\operatorname{arccsc} a\,x + \frac{1}{a}\,\ln\left|a\,x+\sqrt{a^2\,x^2-1}\right|$$

$$\int dx\ x\operatorname{arcsec} a\,x \;=\; \frac{x^2}{2}\operatorname{arcsec} a\,x - \frac{1}{2\,a^2}\,\sqrt{a^2\,x^2-1}$$

$$\int dx\ x\operatorname{arccsc} a\,x \;=\; \frac{x^2}{2}\operatorname{arccsc} a\,x + \frac{1}{2\,a^2}\,\sqrt{a^2\,x^2-1}$$

## W35c.04 **antiderivate di integrandi con esponenziali**

$$\int dx\ \mathsf{e}^{a\,x} \;=\; \frac{1}{a}\,\mathsf{e}^{a\,x} \qquad , \qquad \int dx\,\eta^x \;=\; \int dx\ \mathsf{e}^{x\,\ln\,\eta} \;=\; \frac{\eta^x}{\ln\,b}$$

$$\int dx\ x\,\mathsf{e}^{a\,x} \;=\; \frac{\mathsf{e}^{a\,x}}{a^2}\,(a\,x-1) \qquad , \qquad \int dx\ x^2\,\mathsf{e}^{a\,x} \;=\; \frac{\mathsf{e}^{a\,x}}{a^3}\,(a^2\,x^2-2\,a\,x+2)$$

(1) $$\int dx\ x^n\,\mathsf{e}^{a\,x} \;=$$

$$\frac{\mathsf{e}^{a\,x}}{a^{n+1}}\,\left((a\,x)^n - n\,(a\,x)^{n-1} + n(n-1)(a\,x)^{n-2} - \cdots + (-1)^n\,n!\right) \quad \text{con} \quad n\in\mathbb{P}$$

$$\int \frac{dx}{b+c\,\mathsf{e}^{a\,x}} \;=\; \frac{1}{a\,b}\left(a\,x - \ln|b+c\,\mathsf{e}^{a\,x}|\right)$$

$$\int dx\,\frac{\mathsf{e}^{a\,x}}{b+c\,\mathsf{e}^{a\,x}} \;=\; \frac{1}{a\,b}\,\ln\,|b+c\,\mathsf{e}^{a\,x}|$$

$$\int \frac{\mathrm{d}x}{(b+c\,\mathrm{e}^{a\,x})^2} \;=\; \frac{x}{b^2}+\frac{1}{a\,b(b+c\,\mathrm{e}^{a\,x})}-\frac{1}{a\,b^2}\,\ln|b+c\,\mathrm{e}^{a\,x}|$$

$$\int \mathrm{d}x\,\frac{\mathrm{e}^{a\,x}}{(b+c\,\mathrm{e}^{a\,x})^2} \;=\; -\frac{1}{a\,c(b+c\,\mathrm{e}^{a\,x})}$$

$$\int \mathrm{d}x\;\,x\,\mathrm{e}^{a\,x} \;=\; \frac{1}{2\,a}\,\mathrm{e}^{a\,x}$$

$$\int \mathrm{d}x\;\,x^{2\,n+1}\,\mathrm{e}^{a\,x} \;=\; \lfloor_{\mathbb{N}}\ t:=x^2\ \rfloor_{\mathbb{N}} \;=\; \frac{1}{2}\int \mathrm{d}t\,t^n\,\mathrm{e}^{a\,t}\quad [\mathsf{c04}(1)]$$

$$\int \mathrm{d}x\,\frac{x\,\mathrm{e}^{a\,x}}{(1+a\,x)^2} \;=\; \frac{\mathrm{e}^{a\,x}}{a^2(1+a\,x)}$$

$$\int \mathrm{d}x\;\,\mathrm{e}^{a\,x}\,\sin b\,x \;=\; \frac{\mathrm{e}^{a\,x}}{a^2+b^2}\,(a\,\sin b\,x-b\,\cos b\,x)$$

$$\int \mathrm{d}x\;\,\mathrm{e}^{a\,x}\,\cos b\,x \;=\; \frac{\mathrm{e}^{a\,x}}{a^2+b^2}\,(a\,\cos b\,x+b\,\sin b\,x)$$

$$\int \mathrm{d}x\;\,\mathrm{e}^{a\,x}\,\sin^n b\,x \;=\; \frac{\mathrm{e}^{a\,x}\,\sin^{n-1} b\,x}{a^2+n^2\,b^2}\,(a\,\sin b\,x-n\,b\,\cos b\,x)+\frac{n(n-1)\,b^2}{a^2+n^2\,b^2}\int \mathrm{d}x\;\,\mathrm{e}^{a\,x}\,\sin^{n-2} b\,x$$

$$\int \mathrm{d}x\;\,\mathrm{e}^{a\,x}\,\cos^n b\,x \;=\; \frac{\mathrm{e}^{a\,x}\,\cos^{n-1} b\,x}{a^2+n^2\,b^2}\,(a\,\cos b\,x+n\,b\,\sin b\,x)+\frac{n(n-1)\,b^2}{a^2+n^2\,b^2}\int \mathrm{d}x\;\,\mathrm{e}^{a\,x}\,\cos^{n-2} b\,x$$

$$\int \mathrm{d}x\;\,x\,\mathrm{e}^{a\,x}\,\sin b\,x \;=\; \frac{x\,\mathrm{e}^{a\,x}}{a^2+b^2}\,(a\,\sin b\,x-b\,\cos b\,x)-\frac{\mathrm{e}^{a\,x}}{(a^2+b^2)^2}\,\big((a^2-b^2)\,\sin b\,x-2\,a\,b\,\cos b\,x\big)$$

$$\int \mathrm{d}x\;\,x\,\mathrm{e}^{a\,x}\,\cos b\,x \;=\; \frac{x\,\mathrm{e}^{a\,x}}{a^2+b^2}\,(a\,\cos b\,x+b\,\sin b\,x)-\frac{\mathrm{e}^{a\,x}}{(a^2+b^2)^2}\,\big((a^2-b^2)\,\cos b\,x+2\,a\,b\,\sin b\,x\big)$$

## W35c.05 **antiderivate di integrandi con logaritmi**

$$\int \mathrm{d}x\;\,\ln a\,x \;=\; x\,\ln a\,x-x \qquad , \qquad \int \mathrm{d}x\,(\ln a\,x)^2 \;=\; x\,(\ln a\,x)^2-2\,x\,\ln a\,x+2\,x$$

$$\int \mathrm{d}x\,(\ln a\,x)^n \;=\; x\,(\ln a\,x)^n-n\int \mathrm{d}x\,(\ln a\,x)^{n-1}$$

$$\int \mathrm{d}x\;\,x^n\,(\ln a\,x)^n \;=\; x^{n+1}\left(\frac{\ln a\,x}{n+1}-\frac{1}{(n+1)^2}\right)\quad \text{per}\quad n\neq -1$$

$$\int \mathrm{d}x\,\frac{\ln a\,x}{x} \;=\; \frac{1}{2}\,(\ln a\,x)^2 \qquad , \qquad \int \frac{\mathrm{d}x}{x\,\ln a\,x} \;=\; \ln(\ln a\,x)$$

$$\int \mathrm{d}x\,\frac{(\ln a\,x)^n}{x} \;=\; \frac{(\ln a\,x)^{n+1}}{n+1}\quad \text{per}\quad n\neq -1$$

$$\int \mathrm{d}x\,\frac{\ln a\,x}{x^n} \;=\; \frac{1}{x^{n-1}}\left(\frac{\ln a\,x}{n-1}+\frac{1}{(n-1)^2}\right)\quad \text{per}\quad n\neq -1$$

$$\int \mathrm{d}x\;\,x^n\,(\ln a\,x)^m \;=\; \frac{x^{n+1}}{n+1}\,(\ln a\,x)^m-\frac{m}{n+1}\int \mathrm{d}x\;\,x^n\,(\ln a\,x)^{m-1}\quad \text{per}\quad n\neq -1$$

$$\int \mathrm{d}x\;\,\ln(a\,x+b) \;=\; \frac{a\,x+b}{a}\,\ln(\,a\,x+b)-x$$

$$\int \mathrm{d}x\;\,\ln(x^2+a^2) \;=\; x\,\ln(x^2+a^2)-2\,x+2\,a\,\arctan\frac{x}{a}$$

$$\int \mathrm{d}x \;\; \ln(x^2-a^2) \;=\; x\,\ln(x^2-a^2)-2\,x+a\,\ln\frac{x+a}{x-a}$$

$$\int \mathrm{d}x \;\; x\,\ln(x^2\pm a^2) \;=\; \frac{1}{2}\,(x^2\pm a^2)\,\ln(x^2\pm a^2)-\frac{x^2}{2}$$

$$\int \mathrm{d}x \;\; \ln(\left|x+\sqrt{x^2+a}\right| \;=\; x\,\ln(\left|x+\sqrt{x^2+a}\right|-\sqrt{x^2+a}$$

$$\int \mathrm{d}x \;\; x\,\ln(\left|x+\sqrt{x^2+a}\right| \;=\; \left(\frac{x^2}{2}+\frac{a}{4}\right)\,\ln(\left|x+\sqrt{x^2+a}\right|-\frac{x\sqrt{x^2+a}}{4}$$

$$\int \mathrm{d}x \;\; \sin(\ln\,a\,x) \;=\; \frac{x}{2}\,\big(\sin(\ln\,a\,x)-\cos(\ln\,a\,x)\big)$$

$$\int \mathrm{d}x \;\; \cos(\ln\,a\,x) \;=\; \frac{x}{2}\,\big(\sin(\ln\,a\,x)+\cos(\ln\,a\,x)\big)$$

## W35c.06 **antiderivate di integrandi iperbolici e loro inverse**

$$\int \mathrm{d}x \;\; \sinh a\,x \;=\; \frac{1}{a}\cosh a\,x \quad , \quad \int \mathrm{d}x \;\; \cosh a\,x \;=\; \frac{1}{a}\sinh a\,x$$

$$\int \mathrm{d}x \;\; \tanh a\,x \;=\; \frac{1}{a}\ln(\cosh a\,x) \quad , \quad \int \mathrm{d}x \;\; \coth a\,x \;=\; \frac{1}{a}\ln|\cosh a\,x|$$

$$\int \mathrm{d}x \;\; \sinh^2 a\,x \;=\; \frac{1}{4\,a}\big(\sinh 2\,a\,x-2\,a\,x\big)$$

$$\int \mathrm{d}x \;\; \sinh^n a\,x \;=\; \frac{1}{n\,a}\sinh^{n-1}a\,x\,\cosh a\,x-\frac{n-1}{n}\int \mathrm{d}x \;\; \sinh^{n-2}a\,x$$

$$\int \mathrm{d}x \;\; \operatorname{csch}\,a\,x \;=\; \int \frac{\mathrm{d}x}{\sinh a\,x} \;=\; \frac{1}{a}\ln\left|\tanh\frac{a\,x}{2}\right|$$

$$\int \mathrm{d}x \;\; \operatorname{sech}^2 a\,x \;=\; \int \frac{\mathrm{d}x}{\cosh^2 a\,x} \;=\; \frac{1}{a}\tanh a\,x$$

$$\int \mathrm{d}x \;\; \operatorname{sech} a\,x\,\tanh a\,x \;=\; \int \mathrm{d}x\,\frac{\sinh a\,x}{\cosh^2 a\,x} \;=\; -\frac{1}{a}\operatorname{sech} a\,x$$

$$\int \mathrm{d}x \;\; \cosh^2 a\,x \;=\; \frac{1}{4\,a}\big(\sinh 2\,a\,x+2\,a\,x\big)$$

$$\int \mathrm{d}x \;\; \cosh^n a\,x \;=\; \frac{1}{n\,a}\cosh^{n-1}a\,x\,\sinh a\,x+\frac{n-1}{n}\int \mathrm{d}x \;\; \cosh^{n-2}a\,x$$

$$\int \mathrm{d}x \;\; \operatorname{sech} a\,x \;=\; \int \frac{\mathrm{d}x}{\cosh a\,x} \;=\; \frac{2}{a}\arctan \mathrm{e}^{a\,x}$$

$$\int \mathrm{d}x \;\; \operatorname{csch}^2 a\,x \;=\; \int \frac{\mathrm{d}x}{\sinh^2 a\,x} \;=\; -\frac{1}{a}\coth a\,x$$

$$\int \mathrm{d}x \;\; \operatorname{csch}\,a\,x\,\coth a\,x \;=\; \int \mathrm{d}x\,\frac{\cosh a\,x}{\sinh^2 a\,x} \;=\; -\frac{1}{a}\operatorname{csch}\,a\,x$$

$$\int \mathrm{d}x \;\; \tanh^2 a\,x \;=\; x-\frac{1}{a}\tanh a\,x \quad , \quad \int \mathrm{d}x \;\; \coth^2 a\,x \;=\; x-\frac{1}{a}\coth a\,x$$

$$\int \mathrm{d}x \;\; \operatorname{arsinh} x \;=\; \int \frac{\mathrm{d}x}{\sinh x} \;=\; \int \mathrm{d}x \;\; \ln\left(x+\sqrt{x^2+1}\right) \;=\; x\operatorname{arsinh} x-\sqrt{x^2+1}$$

$$\int \mathrm{d}x \;\; \operatorname{arcosh} x \;=\; \int \frac{\mathrm{d}x}{\cosh x} \;=\; \int \mathrm{d}x \;\; \ln\left(x+\sqrt{x^2-1}\right) \;=\; x\operatorname{arcosh} x-\sqrt{x^2-1}$$

$$\int dx\ \operatorname{artanh} x \ = \ \int \frac{dx}{\tanh\ x} \ = \ x \operatorname{artanh} x + \frac{1}{2}\ \ln(x^2-1)$$

$$\int dx\ \operatorname{arcoth} x \ = \ \int \frac{dx}{\coth\ x} \ = \ x \operatorname{arcoth} x + \frac{1}{2}\ \ln(x^2-1)$$

$$\int \frac{dx}{\operatorname{sech} x} \ = \ \frac{x}{\operatorname{sech} x} + \frac{1}{\sinh\ x} \qquad , \qquad \int \frac{dx}{\operatorname{csch}\ x} \ = \ \frac{x}{\operatorname{csch}\ x} + \frac{\operatorname{sign} x}{\sinh\ x}$$

$$\int dx\, \frac{x}{\operatorname{sech} x} \ = \ \frac{x^2}{2 \operatorname{sech} x} - \frac{1}{2}\sqrt{1-x^2} \qquad , \qquad \int dx\, \frac{x}{\operatorname{csch}\ x} \ = \ \frac{x^2}{2 \operatorname{csch}\ x} + \frac{\operatorname{sign} x}{2}\sqrt{1-x^2}$$

$$\int dx\, \frac{1}{(\sinh\ x)^2} \ = \ \tanh\ x \qquad , \qquad \int dx\, \frac{1}{(\cosh\ x)^2} \ = \ \coth\ x$$

## W35 d. integrali definiti

Ricordiamo la costante di Euler-Mascheroni $\gamma_{em} = 0.57721\,56649$ [I13d03]

e la funzione Gamma $\Gamma(z) \;:=\; \lim_{n\to+\infty} \dfrac{n^z\,n!}{z(z+1)(z+2)\cdots(z+n)}$ [W60d01]

### W35d.01 integrali definiti di integrandi algebrici

$$\int_a^b \mathrm{d}x\ x^{m-1}\,(1-x)^{n-1} \;=\; \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} \quad \text{per} \quad m,n>0$$

$$\int_0^1 \mathrm{d}x\,(x-a)^{m-1}\,(b-x)^{n-1} \;=\; (b-a)^{m+n-1}\,\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} \quad \text{per} \quad m,n>0\ ,\ a<b$$

$$\int_0^1 \mathrm{d}x\,\frac{x^n}{1+x} \;=\; (-1)^n\left(\ln 2 - 1 + \frac{1}{2} - \cdots + \frac{(-1)^n}{n}\right) \quad \text{per} \quad n\in\mathbb{P}$$

$$\int_0^1 \frac{\mathrm{d}x}{(1-x)^{1/n}} \;=\; \frac{\pi}{n\,\sin\frac{\pi}{n}} \quad \text{per} \quad n>1$$

$$\int_0^1 \mathrm{d}x\,\frac{x^a}{\sqrt{1-x^2}} \;=\; \frac{\sqrt{\pi}\,\Gamma\left(\frac{a+1}{2}\right)}{2\,\Gamma\left(\frac{a+2}{2}\right)} \quad \text{per} \quad -1<a$$

$$\int_0^1 \mathrm{d}x\,\frac{x^{a-1}}{(1-a)^a} \;=\; \frac{\pi}{\sin\,a\,\pi} \quad \text{per} \quad 0<a<1$$

$$\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^a}} \;=\; \frac{\sqrt{\pi}\,\Gamma(1/a)}{a\,\Gamma\left(\frac{1}{a}+\frac{1}{2}\right)}$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{1+x^a} \;=\; \frac{\pi}{a\,\sin\frac{x}{a}} \quad \text{per} \quad a>1$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{x^a\,(1+x)} \;=\; \frac{\pi}{b\,\sin\,a\,x} \quad \text{per} \quad 0<a<1$$

$$\int_0^{+\infty} \mathrm{d}x\,\frac{x^{a-1}}{1+x^b} \;=\; \frac{\pi}{b\,\sin\left(\frac{a\,\pi}{b}\right)} \quad \text{per} \quad 0<a<b$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{a^2+x^2} \;=\; \frac{\pi}{2\,a} \quad \text{per} \quad 0<a$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{(a^2+x^2)^n} \;=\; \frac{\pi\,(2\,n-3)!!}{2\,a^{2\,n-1}\,(2\,n-2)!!} \quad \text{per} \quad 0<a\ ,\ n=2,3,4....$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{(a^2+x^2)\,(b^2+x^2)} \;=\; \frac{\pi}{2\,a\,b\,(a+b)} \quad \text{per} \quad a,b>0$$

$$\int_0^{+\infty} \mathrm{d}x\,\frac{x^{m-1}}{(a\,x+b)^{m+n}} \;=\; \frac{\Gamma(m)\,\Gamma(n)}{a^m\,b^n\,\Gamma(m+n)} \quad \text{per} \quad a,b,m,n>0$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{a\,x^2+2\,b\,x+c} \;=\; \frac{1}{\sqrt{a\,c-b^2}}\left(\frac{\pi}{2} - \arctan\,\frac{b}{\sqrt{a\,c-b^2}}\right) \quad \text{per} \quad a,a\,c-b^2>0$$

$$\int_0^{+\infty} \frac{\mathrm{d}x}{a\,x^2+2\,b\,x+c} \;=\; \frac{\pi}{2\,\sqrt{c\,d}} \quad \text{ove } d \;:=\; 2(b+\sqrt{a\,c}) \quad \text{per} \quad a,c,d>0$$

$$\int_1^{+\infty} \mathrm{d}x\,\left(\frac{1}{\lfloor x\rfloor} - \frac{1}{x}\right) \;=\; \gamma_{em}\ \text{[I13d03]}$$

## W35d.02 integrali definiti di integrandi esponenziali

Consideriamo $a \in \mathbb{R}_+$ , $n \in \mathbb{N}$ , $h \in \mathbb{P}$ .

$$\int_0^{+\infty} dx\, x^c\, e^{-a\,x} = \frac{\Gamma(c+1)}{a^{c+1}} \quad \text{per} \quad c \in (-1, +\infty)$$

$$\int_0^{+\infty} dx\, \sqrt{x}\, e^{-a\,x} = \frac{1}{2\,a}\sqrt{\pi}$$

$$\int_0^{+\infty} dx\, x^n\, e^{-x} = n! \quad \text{per} \quad n \in \mathbb{N}$$

$$\int_0^{+\infty} dx\, x^n\, e^{-a\,x} = \frac{n!}{a^{n+1}} \quad \text{per} \quad n \in \mathbb{N}$$

$$\int_0^{+\infty} dx\, e^{-a\,x^2} = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{+\infty} dx\, x^2\, e^{-a\,x^2} = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^{+\infty} dx\, x^{2\,h}\, e^{-a\,x^2} = \frac{2\,n-1}{2\,a}\int_0^{+\infty} dx\, x^{2\,h}\, e^{-a\,x^2} = \frac{(2\,h-1)!!}{2^{h+1}}\sqrt{\frac{\pi}{a^{2\,h+1}}} = \frac{(2\,h)!}{h!\,2^{2\,h+1}\,2^{h+1}}\sqrt{\frac{\pi}{a^{2\,h+1}}}$$

$$\int_0^{+\infty} dx\, x^c\, e^{-a\,x^2} = \frac{1}{2}\,\frac{\Gamma((c+1)/2)}{a^{(c+1)/2}}$$

$$\int_0^{+\infty} dx\, x^{2\,h}\, e^{-a\,x^2} = \frac{(2h-1)!!}{2^{h+1}\,a^h}\sqrt{\frac{\pi}{a}} \quad \text{per} \quad h \in \mathbb{N}$$

$$\int_0^{+\infty} dx\, x^{2\,h+1}\, e^{-a\,x^2} = \frac{h!}{2\,a^{h+1}} \quad \text{per} \quad h \in \mathbb{N}$$

$$\int_{-\infty}^{+\infty} dx\, e^{2\,b\,x-a\,x^2} = \sqrt{\frac{\pi}{a}}\, e^{b^2/a} \quad \text{per} \quad a > 0\ ,\ b \in \mathbb{R}$$

$$\int_0^1 dx\, x^{-x} = \int_0^1 dx\, e^{-x\,\ln x} = \sum_{n=1}^{+\infty}\frac{1}{n^n} \approx 1.29128\ 59970\ 62664$$

$$\int_0^1 dx\, x^x = \int_0^1 dx\, e^{x\,\ln x} = -\sum_{n=1}^{+\infty}\frac{(-1)^{n+1}}{n^n} = -\sum_{n=1}^{+\infty}\frac{1}{(-n)^n} \approx 0.78343\ 05107\ 12134$$

## W35d.03 integrali definiti di integrandi logaritmici

$$\int_0^1 dx\, (\ln x)^n = (-1)^n\, n! \quad \text{per} \quad n \in \mathbb{P}$$

$$\int_0^1 dx\, \ln|\ln x| = \int_0^{+\infty} dx\, e^{-x}\,\ln x = \gamma_{em}$$

$$\int_0^1 dx\, \frac{\ln x}{x-1} = \frac{\pi^2}{6}$$

$$\int_0^1 dx\, \frac{\ln x}{x+1} = -\frac{\pi^2}{12}$$

$$\int_0^1 \mathrm{d}x\ \frac{\ln\,x}{\sqrt{1-x^2}} \;=\; -\frac{\pi}{2}\,\ln\,2$$

## W35d.04 integrali definiti di integrandi trigonometrici

$$\int_0^{\pi/2} \mathrm{d}x\ \sin^n\,x \;=\; \int_0^{\pi/2} \mathrm{d}x\ \cos^n\,x \;=\; \begin{cases} \frac{(n-1)!!}{n!!} & \text{per } n=1,3,5,... \\ \frac{(n-1)!!}{n!!}\,\frac{\pi}{2} & \text{per } n=2,4,6,... \end{cases}$$

$$\int_0^{\pi/2} \mathrm{d}x\ \sin^a\,x \;=\; \int_0^{\pi/2} \mathrm{d}x\ \cos^a\,x \;=\; \frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\frac{a+1}{2}\right)}{\Gamma\left(\frac{a+2}{2}\right)} \quad \text{per} \quad a>-1$$

$$\int_0^{\pi} \mathrm{d}x\ x\,\sin^n\,x \;=\; \begin{cases} \frac{(n-1)!!}{n!!}\,\pi & \text{per } n=1,3,5,... \\ \frac{(n-1)!!}{n!!}\,\frac{\pi^2}{2} & \text{per } n=2,4,6,... \\ \frac{n^{3/2}}{2}\,\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)} & \text{per } n>-1 \end{cases}$$

$$\int_0^{\pi/2} \mathrm{d}x\ \sin^{2\,a+1}\,x\ \cos^{2\,b+1}\,x \;=\; \frac{\Gamma(a+1)\,\Gamma(b+1)}{2\,\Gamma(a+b+2)}$$

Consideriamo gli interi $m$ ed $n$.

$$\int_0^{\pi} \mathrm{d}x\ \sin\,m\,x\ \sin\,n\,x \;=\; \begin{cases} 0 & \text{per } m\neq n \\ \frac{\pi}{2} & \text{per } m=n \end{cases}$$

$$\int_0^{\pi} \mathrm{d}x\ \cos\,m\,x\ \cos\,n\,x \;=\; \begin{cases} 0 & \text{per } m\neq n \\ \frac{\pi}{2} & \text{per } m=n\neq 0 \\ \pi & \text{per } m=n=0 \end{cases}$$

$$\int_0^{\pi} \mathrm{d}x\ \sin\,m\,x\ \cos\,n\,x \;=\; \begin{cases} 0 & \text{per } m+n \text{ pari} \\ \frac{2\,m}{m^2-n^2} & \text{per } m+n \text{ dispari} \end{cases}$$

$$\int_0^{\pi/2} \frac{\mathrm{d}x}{1+a\,\cos\,x} \;=\; \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{1+a\,\sin\,x} \;=\; \frac{\arccos a}{\sqrt{1-a^2}} \quad \text{per} \quad |a|<1$$

$$\int_0^{\pi} \frac{\mathrm{d}x}{1+a\,\sin\,x} \;=\; \frac{2\,a\arccos a}{\sqrt{1-a^2}} \quad \text{per} \quad -1<a<1$$

$$\int_0^{\pi} \frac{\mathrm{d}x}{1+a\,\cos\,x} \;=\; \frac{\pi}{\sqrt{1-a^2}} \quad \text{per} \quad -1<a<1$$

$$\int_0^{\pi/2} \frac{\mathrm{d}x}{a^2\,\cos^2\,x+b^2\,\sin^2\,x} \;=\; \frac{\pi}{2\,a\,b} \quad \text{per} \quad a,b>0$$

$$\int_0^{+\infty} \mathrm{d}x\ \sin\,x^2 \;=\; \int_0^{+\infty} \mathrm{d}x\ \cos\,x^2 \;=\; \frac{\sqrt{2\,\pi}}{4}$$

$$\int_0^{+\infty} \mathrm{d}x\ \sin\,x^a \;=\; \Gamma\left(1+\frac{1}{a}\right)\,\sin\,\frac{\pi}{2\,a}$$

$$\int_0^{+\infty} \mathrm{d}x\ \cos\,x^a \;=\; \Gamma\left(1+\frac{1}{a}\right)\,\cos\,\frac{\pi}{2\,a}$$

$$\int_0^{+\infty} \mathrm{d}x\ \frac{\sin\,a\,x}{x} \;=\; \frac{\pi}{2} \quad \text{per} \quad a>0$$

$$\int_0^{+\infty} \mathrm{d}x\ \frac{\sin\,x}{\sqrt{x}} \;=\; \int_0^{+\infty} \mathrm{d}x\ \frac{\cos\,x}{\sqrt{x}} \;=\; \sqrt{\frac{\pi}{2}}$$

$$\int_0^{+\infty} \mathrm{d}x\, \frac{\sin^3 x}{x^3} = \frac{3\pi}{8}$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\sin^4 x}{x^4} = \frac{\pi}{3}$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\sin x}{x^a} = \frac{\pi}{2\,\Gamma(a)\,\sin(a\,\pi/2)} \quad \text{per } 0 < a < 2$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\cos x}{x^a} = \frac{\pi}{2\,\Gamma(a)\,\cos(a\,\pi/2)} \quad \text{per } 0 < a < 1$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\cos a\,x - \cos b\,x}{x} = \ln\frac{b}{a}$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{x\,\sin a\,x}{b^2+x^2} = \frac{\pi}{2}\,\mathrm{e}^{-a\,b} \quad \text{per } a,b > 0$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\cos a\,x}{b^2+x^2} = \frac{\pi}{2\,b}\,\mathrm{e}^{-a\,b} \quad \text{per } a,b > 0$$

### W35**d.05** integrali definiti di integrandi espotrigonometrici e logtrigonometrici

Consideriamo $a > 0$ , $n \in \mathbb{N}$ , $h \in \mathbb{P}$ .

$$\int_0^{+\infty} \mathrm{d}x\, \mathrm{e}^{-a\,x}\,\sin b\,x = \frac{b}{a^2+b^2} \quad , \quad \int_0^{+\infty} \mathrm{d}x\, \mathrm{e}^{-ax}\,\sin b\,x = \frac{a}{a^2+b^2}$$
$$\int_0^{+\infty} \mathrm{d}x\, x\,\mathrm{e}^{-a\,x}\,\sin b\,x = \frac{2\,a\,b}{(a^2+b^2)^2} \quad , \quad \int_0^{+\infty} \mathrm{d}x\, x\,\mathrm{e}^{-a\,x}\,\cos b\,x = \frac{a^2-b^2}{a^2+b^2}$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\mathrm{e}^{-a\,x}\,\sin b\,x}{x} = \arctan\frac{b}{a} \quad \text{per} \quad a > 0$$
$$\int_0^{\frac{\pi}{2}} \mathrm{d}x\ \ln(\sin x) = \int_0^{\frac{\pi}{2}} \mathrm{d}x\ \ln(\cos x) = -\frac{\pi}{2}\ln 2$$
$$\int_0^{+\infty} \mathrm{d}x\ \int_z^{\frac{\pi}{4}} \mathrm{d}x\ \ln(1+\tan x) = \frac{\pi}{8}\ln 2$$
$$\int_0^{+\infty} \mathrm{d}x\, \frac{\sin x}{x}\,\ln x = -\frac{\pi}{2}\,\gamma_{em}$$

`L'esposizione in https://www.mi.imati.cnr.it/alberto/ e https://arm.mi.imati.cnr.it/Matexp/matexp_main.php`