1

Capitolo W30

prontuario: limiti, derivate, serie

Contenuti delle sezioni

a. limiti p. 2
b. derivate p. 3
d. serie numeriche p. 5
f. sviluppi in serie di potenze p. 6

8 pagine

W30 **a. limiti**

W30a.01

$$\lim_{x\to0}\frac{\sin x}{x}=1 \quad , \quad \lim_{x\to0}\frac{\tan x}{x}=1 \quad , \quad \lim_{x\to0}\frac{1-\cos x}{x}=0 \quad , \quad \lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$\mathsf{e}:=\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n \approx 2.71828\,18284\,59045 \quad , \quad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\mathsf{e}$$

$$\forall\alpha\in\mathbb{R}, m\in\mathbb{N}\ :\ \lim_{x\to\infty}\left(1+\frac{\alpha}{x}\right)^{m\,x}=\mathsf{e}^{\alpha\,m} \quad \text{in partic. } \lim_{x\to\infty}\left(1+\frac{\alpha}{x}\right)^x=\mathsf{e}^{\alpha} \quad , \quad \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x=\frac{1}{\mathsf{e}}$$

$$\lim_{x\to0}(1+\alpha\,x)^{\frac{1}{x}}=\mathsf{e}^{\alpha} \quad \text{in partic. } \lim_{x\to0}(1+x)^{\frac{1}{x}}=\mathsf{e} \quad , \quad \lim_{x\to0}(1-x)^{\frac{1}{x}}=\frac{1}{\mathsf{e}} \quad , \quad \lim_{x\to0}\frac{\ln(1+x)}{x}=1$$

$$\forall a\in\mathbb{R}_+ :\ \lim_{x\to0}\frac{a^x-1}{x}=\ln a \quad , \quad \lim_{x\to0}\frac{\mathsf{e}^x-1}{x}=1 \quad , \quad \lim_{x\to0}\frac{(1+x)^{\alpha}-1}{x}=\alpha$$

$$\forall p\in\mathbb{R}_+ :\ \lim_{x\to+\infty}\frac{\ln x}{x^p}=0 \quad , \quad \lim_{x\to0+}x^p\,\ln x=0 \quad , \quad \lim_{x\to+\infty}(x-\alpha\,\ln x)=+\infty$$

$$\forall M\in\mathbb{R} :\ \lim_{x\to+\infty}\frac{\mathsf{e}^x}{x^M}=+\infty \quad , \quad \lim_{x\to+\infty}x^M\,\mathsf{e}^{-x}=0$$

$$\gamma_{em} := \lim_{n\to+\infty}\left(\sum_{j=1}^{n}\frac{1}{j}-\ln n\right) \approx = 0.57721\,56649\,01532$$ **costante di Euler-Mascheroni**

W30 **b. derivate**

W30b.01 Una $f(x)$ funzione definita in un intervallo reale I a valori reali si dice **funzione derivabile nell'intervallo** I sse

$$\forall x \in I \ : \ \mathrm{D}_x(f(x)) := \frac{\mathrm{d}f}{\mathrm{d}x}(x) := f'(x) := \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =_{\text{if}\exists} \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

Esempi:

$$\forall \alpha \in \mathbb{R} \ : \ \frac{\mathrm{d}}{\mathrm{d}x}(x^\alpha) = \alpha x^{\alpha-1} \ . \quad \text{in partic.} \ \ \forall \mathtt{C} \in \mathbb{R} : \frac{\mathrm{d}}{\mathrm{d}x}\mathtt{C} = 0 \quad , \quad \frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x} = -\frac{1}{x^2} \quad , \quad \forall n = 0,1,2,\ldots : \frac{\mathrm{d}}{\mathrm{d}x}\sqrt[n]{x} = \frac{1}{n}\sqrt[n]{x^{n-1}} \quad , \quad \forall \beta \in \mathbb{R}_+ : \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x^\beta} = -\frac{\beta x^{\beta-1}}{x^{2\beta}} = -\beta x^{-\beta-1}$$

$$\forall \beta \in \mathbb{R}_+ \setminus \{1\} \ : \ \frac{\mathrm{d}}{\mathrm{d}x}(\beta^x) = \beta^x \ln \beta \ . \quad \text{in partic.} \ \frac{\mathrm{d}}{\mathrm{d}x}(\mathsf{e}^x) = \mathsf{e}^x \ , \ \frac{\mathrm{d}}{\mathrm{d}x}(\mathsf{e}^{-x}) = -\mathsf{e}^{-x} \ , \ \frac{\mathrm{d}}{\mathrm{d}x}(\mathsf{e}^{\beta x}) = \beta\, \mathsf{e}^{\beta x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\log_\beta x) = \frac{1}{x} \ \log_\beta \mathsf{e} = \frac{1}{\ln \beta \, x} \quad , \quad \frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = \mathrm{D}_x(\log_{\mathsf{e}} x) = \frac{1}{x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x \quad , \quad \frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = -\sin x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec x = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{\cos x}\right] = \frac{\sin x}{\cos^2 x} = \frac{\tan x}{\cos x} \quad , \quad \frac{\mathrm{d}}{\mathrm{d}x}\csc x = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{\sin x}\right] = -\frac{\cos x}{\sin^2 x} = \frac{\cot x}{\sin x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot x = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\cos x}{\sin x}\right] = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -1 - \cot^2 x = -\frac{1}{\sin^2 x}$$

$$y = \arcsin x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\arcsin x = \frac{1}{\mathrm{D}_y \sin y} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

$$y = \arccos x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\arccos x = \frac{1}{\mathrm{D}_y \cos y} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 x}} = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \arctan x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\arctan x = \frac{1}{\mathrm{D}_y \tan y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

$$y = \operatorname{arccot} x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arccot} x = \frac{1}{\mathrm{D}_y \cot y} = \frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x \quad , \quad \frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = -\sinh x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tanh x = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\sinh x}{\cosh x}\right] = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 - \tanh^2 x = \frac{1}{\cosh^2 x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\coth x = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\cosh x}{\sinh x}\right] = 1 - \coth^2 x = -\frac{1}{\sin^2 x}$$

$$y = \operatorname{arsinh} x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arsinh} x = \frac{1}{\mathrm{D}_y \sinh y} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+x^2}} \ .$$

$$y = \operatorname{arcosh} x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arcosh} x = \frac{1}{\mathrm{D}_y \cosh y} = -\frac{1}{\sinh y} = -\frac{1}{\sqrt{1-\cosh^2 x}} = -\frac{1}{\sqrt{x^2-1}}$$

$$y = \operatorname{artanh} x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\operatorname{artanh} x = \frac{1}{\mathrm{D}_y \tanh y} = \frac{1}{1+\tanh^2 y} = \frac{1}{1+x^2}$$

$$y = \operatorname{arcoth} x \ : \ \frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arcoth} x = \frac{1}{\mathrm{D}_y \coth y} = \frac{1}{1-x^2}$$

W30b.02 **regole di derivazione**

$f(x)$ e $g(x)$ derivabili in x $\quad \mathrm{D}_x\big(\alpha\, f(x) + \beta\, g(x)\big) \;=\; \alpha\, \mathrm{D}_x f(x) + \beta\, \mathrm{D}_x g'(x)$

$$\mathrm{D}_x\big(f(x)\cdot g(x)\big) \;=\; \mathrm{D}_x f(x)\cdot g(x) + f(x)\cdot \mathrm{D}_x g(x) \quad , \quad \mathrm{D}_x\left(\frac{f(x)}{g(x)}\right) \;=\; \frac{f'(x)\, g(x) - f(x)\, g'(x)}{g(x)}$$

$$\mathrm{D}_x f(g(x)) \;=\; \mathrm{D}_{\,g(x)}(g(x))\cdot \mathrm{D}_x g(x) \quad , \quad \mathrm{D}_x f\big(g(h(x))\big) \;=\; \mathrm{D}_{\,g}\big(f(h(x))\big)\cdot \mathrm{D}_{\,h}\big(g(h(x)\big)\cdot \mathrm{D}_x h(x)$$

$$\mathrm{D}_x \mathsf{e}^{f(x)} \;=\; \mathsf{e}^{f(x)}\cdot \mathrm{D}_x f(x) \quad , \quad \mathrm{D}_x \ln|f(x)| \;=\; \frac{\mathrm{D}_x f(x)}{f(x)}$$

$$\mathrm{D}_x[f(x)]^{g(x)} \;=\; [f(x)]^{g(x)}\left[g'(x)\cdot \ln f(x) + \frac{g(x)\, f'(x)}{f(x)}\right]$$

W30 **d. serie numeriche**

W30**d.01**

$$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}+\cdots \;=\; \mathsf{e} \quad , \quad 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^n\,\frac{1}{n!}\pm\cdots \;=\; \frac{1}{\mathsf{e}}$$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots+(-1)^{n-1}\frac{1}{n}+\cdots \;=\; \ln 2 \quad , \quad 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots \;=\; 2$$

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots+\frac{(-1)^n}{2^n}+\cdots \;=\; \frac{2}{3} \quad , \quad 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots-\frac{1}{4n-1}+\frac{1}{4n+1}-\cdots \;=\; \frac{\pi}{4}$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}+\cdots=1 \quad , \quad 1+\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}+\cdots=\frac{1}{2}$$

$$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{(n-1)(n+1)}+\cdots=\frac{3}{4}$$

$$\frac{1}{3\cdot 5}+\frac{1}{7\cdot 9}+\frac{1}{11\cdot 13}+\cdots+\frac{1}{(4n-1)(4n+1)}+\cdots=\frac{1}{2}-\frac{\pi}{8}$$

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\cdots+\frac{1}{n(n+1)(n+2)}+\cdots=\frac{1}{4}$$

$$\frac{1}{1\cdot 2\cdots h}+\frac{1}{2\cdot 3\cdots(h+1)}+\frac{1}{3\cdot 4\cdots(h+2)}+\cdots+\frac{1}{n(n+1)\cdots(n+h-1)}+\cdots=\frac{1}{(h-1)\cdot(h-1)!}$$

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots+\frac{1}{n^2}+\cdots \;=\; \frac{\pi^2}{6} \quad , \quad 1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots+(-1)^{n-1}\frac{1}{n^2}+\cdots \;=\; \frac{\pi^2}{12}$$

$$1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots+\frac{1}{n^4}+\cdots \;=\; \frac{\pi^4}{90} \quad , \quad 1-\frac{1}{2^4}+\frac{1}{3^4}-\frac{1}{4^4}+\cdots+(-1)^{n+1}\frac{1}{n^4}+\cdots \;=\; \frac{7\,\pi^4}{720}$$

$$1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots+\frac{1}{n^4}+\cdots \;=\; \frac{\pi^4}{90} \quad , \quad 1-\frac{1}{2^4}+\frac{1}{3^4}-\frac{1}{4^4}+\cdots+(-1)^{n+1}\frac{1}{n^4}+\cdots \;=\; \frac{7\,\pi^4}{720}$$

$$1+\frac{1}{3^2}+\frac{1}{5^2}+-\frac{1}{7^2}+\cdots+\frac{1}{(2n+1)^2}+\cdots \;=\; \frac{\pi^2}{8} \quad , \quad 1+\frac{1}{3^4}+\frac{1}{5^4}+-\frac{1}{7^4}+\cdots+\frac{1}{(2n+1)^4}+\cdots \;=\; \frac{\pi^4}{96}$$

W30**d.02** serie per numeri di Bernoulli e di Eulero

$$1+\frac{1}{2^{2k}}+\frac{1}{3^{2k}}+\frac{1}{4^{2k}}+\cdots+\frac{1}{n^{2k}}+\cdots \;=\; \frac{\pi^{2k}\,2^{2k-1}}{(2k)!}\,\mathrm{Brn}_k$$

$$1-\frac{1}{2^{2k}}+\frac{1}{3^{2k}}-\frac{1}{4^{2k}}+\cdots+(-1)^{n-1}\,\frac{1}{n^{2k}}+\cdots \;=\; \frac{\pi^{2k}\,(2^{2k-1}-1)}{(2k)!}\,\mathrm{Brn}_k$$

$$1+\frac{1}{3^{2k}}+\frac{1}{5^{2k}}+\frac{1}{7^{2k}}+\cdots+(-1)^{n-1}\,\frac{1}{(2n-1)^{2k}}+\cdots \;=\; \frac{\pi^{2k}\,(2^{2k-1}-1)}{2\,(2k)!}\,\mathrm{Brn}_k$$

$$1-\frac{1}{3^{2k+1}}+\frac{1}{5^{2k+1}}-\frac{1}{7^{2k+1}}+\cdots+(-1)^{n-1}\,\frac{1}{(2n-1)^{2k+1}}+\cdots \;=\; \frac{\pi^{2k}\,(2^{2k-1}-1)}{2\,(2k)!}\,\mathrm{Eul}_k$$

W30 f. sviluppi in serie di potenze

In questa sezione assumiamo che sia $\alpha \in \mathbb{R}$ e $a, b \in \mathbb{R}_{nz}$

W30f.01 sviluppi in serie di potenze di espressioni algebriche

$$(1+x)^\alpha = 1 + \alpha\, x + \frac{\alpha(\alpha-1)}{2}\, x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6}\, x^3 \cdots + \binom{a}{n} x^n + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n\, x^n + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\frac{1}{a - b\,x} = \frac{1}{a}\left(1 + \frac{b\,x}{a} + \left(\frac{b\,x}{a}\right)^2 + \cdots + \left(\frac{b\,x}{a}\right)^n + \cdots\right) \quad \text{per} \quad |x| < \frac{|a|}{|b|}$$

$$= -\frac{1}{b\,x}\left(1 + \frac{a}{b\,x} + \left(\frac{a}{b\,x}\right)^2 + \cdots + \left(\frac{a}{b\,x}\right)^n + \cdots\right) \quad \text{per} \quad |x| > \frac{|a|}{|b|}$$

$$\frac{1}{(1-x)^2} = 1 + 2\,x + 3\,x^2 + \cdots + (n+1)\,x^n + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5\,x^4}{128} + \cdots + +\binom{1/2}{n} x^n \quad \text{per} \quad -1 < x < 1$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3\,x^2}{8} - \frac{5, x^3}{16} + \frac{35\,x^4}{128} - \cdots + \binom{-1/2}{n} x^n + \cdots \quad \text{per} \quad -1 < x \leq 1$$

W30f.02 sviluppi in serie di potenze per esponenziali

$$\mathsf{e}^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \text{ ‘ per} \quad -\infty < x < +\infty$$

$$a^x = 1 + x \ln a + \frac{(x \ln a)^2}{2} + \frac{(x \ln a)^3}{3!} + \cdots + \frac{(x \ln a)^n}{n!} + \cdots \quad \text{per} \quad -\infty < x < +\infty$$

$$\frac{1}{\mathsf{e}^x - 1} = \frac{1}{x} - \frac{1}{2} + \frac{x}{12} - \frac{x^3}{30\,4!} + \cdots + \frac{\mathrm{Brn}_{2n}\, x^{2\,n-1}\, n}{(2\,n)!} + \cdots \quad \text{per} \ -2\,\pi < x < 0 \ \text{ e } \ 0 < x < 2\,\pi$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2\,n+1}}{(2\,n+1)!} + \cdots \quad \text{per} \quad -\infty < x < +\infty$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2\,n}}{(2\,n)!} + \cdots \quad \text{per} \quad -\infty < x < +\infty$$

$$\tanh x = x - \frac{x^3}{3!} + \frac{2\,x^5}{15} - \frac{17\,x^7}{315} + \cdots + \frac{2^{2n}(2^{2n}-1)}{(2n)!} \mathrm{Brn}_{2n}\, x^{2\,n-1} + \cdots \quad \text{per} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2\,x^5}{945} + \cdots + \frac{2^{2n}}{(2n)!} \mathrm{Brn}_{2n}\, x^{2\,n-1} + \cdots \quad \text{per} \quad \pi < x < 0 \ \text{ e } \ 0 < x < \pi$$

$$\frac{1}{\sinh\, x} \;=\; \frac{1}{x} - \frac{x}{6} + \frac{7\,x^3}{360} + \cdots + \frac{2^{2n}-2}{(2n)!}\,\mathrm{Brn}_{2n}\,x^{2\,n-1} + \cdots \quad \text{per} \quad \pi < x < 0 \;\; \text{e} \;\; 0 < x < \pi$$

$$\mathrm{csch}\;x = \frac{1}{\cosh\,x} \;=\; 1 - \frac{x^2}{2} + \frac{5\,x^4}{24} + \cdots + \frac{\mathrm{Eul}_{2n}}{(2\,n)!}\,x^{2\,n} + \cdots \quad \text{per} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\ln(1+x) \;=\; x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\,\frac{x^n}{n} + \cdots \quad \text{per} \quad -1 < x \le 1$$

$$\ln(a+x) \;=\; \ln\,a + \frac{x}{a} - \frac{1}{2}\left(\frac{x}{a}\right)^2 + \frac{1}{3}\left(\frac{x}{a}\right)^3 - \cdots + (-1)^{n-1}\,\frac{1}{n}\left(\frac{x}{a}\right)^n + \cdots \quad \text{per} \quad -a < x \le a$$

$$\ln(1+x) \;=\; \frac{x}{1+x} + \frac{1}{2}\left(\frac{x}{1+x}\right)^2 + \cdots + \frac{1}{n}\left(\frac{x}{1+x}\right)^n + \cdots \quad \text{per} \quad -\frac{1}{2} < x$$

$$\mathrm{arsinh}\,x \;=\; x - \frac{x^3}{6} + \frac{3\,x^5}{40} - \cdots + (-1)^n\,\frac{(2n-1)!!}{(2\,n)!!\,(2\,n+1)}\,x^{2\,n+1} + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\mathrm{arcosh}\,x \;=\; \ln|2\,x| - \frac{1}{4\,x^2} - \frac{3}{32\,x^4} - \cdots - \frac{(2\,n-1)!!}{(2\,n)!!\,2\,n\,x^{2\,n}} - \cdots \quad \text{per} \quad |x| > 1$$

$$\mathrm{artanh}\,x \;=\; x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2\,n+1}}{2\,n+1} + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\mathrm{arcoth}\,x \;=\; \frac{1}{x} + \frac{1}{3\,x^3} + \frac{1}{5\,x^5} + \cdots + \frac{1}{(2\,n+1)\,5\,x^{2\,n+1}} \quad \text{per} \quad |x| > 1$$

W30f.03 **sviluppi in serie di potenze per espressioni trigonometriche**

$$\sin\,x \;=\; x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n\,\frac{x^{2\,n+1}}{(2\,n+1)!} + \cdots \quad \text{per} \quad -\infty < x < +\infty$$

$$\cos\,x \;=\; 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n\,\frac{x^{2\,n}}{(2\,n)!} + \cdots \quad \text{per} \quad -\infty < x < +\infty$$

$$\tan\,x \;=\; x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \frac{17\,x^7}{315} + \cdots + (-1)^{n-1}\,\frac{2^{2n}(2^{2n}-1)}{(2n)!}\,\mathrm{Brn}_{2n}\,x^{2\,n-1} + \cdots \quad \text{per} \quad \frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\cot\,x \;=\; \frac{1}{x} - \frac{x}{3} + \frac{x^3}{45} + \frac{2\,x^5}{945} - \cdots + (-1)^n\,\frac{2^{2n}}{(2n)!}\,\mathrm{Brn}_{2n}\,x^{2\,n-1} + \cdots \quad \text{per} \quad \pi < x < 0 \;\; \text{e} \;\; 0 < x < \pi$$

$$\sec x = \frac{1}{\cos\,x} \;=\; 1 - \frac{x^2}{2} + \frac{5\,x^4}{24} + \frac{61\,x^6}{6!} + \cdots + (-1)^n\,\frac{\mathrm{Eul}_{2n}}{(2\,n)!}\,x^{2\,n} + \cdots \quad \text{per} \quad \frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\csc\,x \;=\; \frac{1}{x} + \frac{x}{6} + \frac{7\,x^3}{360} + \frac{31\,x^5}{3\,7!} + \cdots + (-1)^{n-1}\,\frac{2^{2n}-2}{(2n)!}\,\mathrm{Brn}_{2n}\,x^{2\,n-1} + \cdots \quad \text{per} \quad \pi < x < \pi\;,\; x \ne 0$$

$$\arcsin\,x \;=\; x + \frac{x^3}{6} + \frac{3,x^5}{40} - + \frac{(2\,n1)!!}{(2\,n)!!\,(2\,n+1)}\,x^{2\,n+1} + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\arctan x \;=\; x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + +(-1)^n \, \frac{x^{2\,n+1}}{2\,n+1} + \cdots \quad \text{per} \quad -1 < x < 1$$

$$\arccos x \;=\; \frac{\pi}{2} - \arcsin x \quad \text{per} \quad -1 < x < 1$$

$$\operatorname{arccot} x \;=\; \frac{\pi}{2} - \arctan x \quad \text{per} \quad -1 < x \leq 1$$

`L'esposizione in https://www.mi.imati.cnr.it/alberto/ e https://arm.mi.imati.cnr.it/Matexp/matexp_main.php`