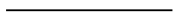


Capitolo W30:
prontuario: limiti, derivate, serie

Contenuti delle sezioni

- a. limiti p.2
- b. derivate p.3
- d. serie numeriche p.5
- e. successioni e serie di funzioni p.6
- f. sviluppi in serie di potenze p.7
- g. prodotti infiniti p.10

10 pagine



W30:a. limiti

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad , \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad , \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad , \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$e := \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n \approx 2.71828\ 18284\ 59045 \quad , \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\forall \alpha \in \mathbb{R}, m \in \mathbb{N} : \lim_{x \rightarrow \infty} \left(1 + \frac{\alpha}{x}\right)^{mx} = e^{\alpha m} \quad \text{in partic.} \quad \lim_{x \rightarrow \infty} \left(1 + \frac{\alpha}{x}\right)^x = e^\alpha \quad , \quad \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x = \frac{1}{e}$$

$$\lim_{x \rightarrow 0} (1 + \alpha x)^{\frac{1}{x}} = e^\alpha \quad \text{in partic.} \quad \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e \quad , \quad \lim_{x \rightarrow 0} (1 - x)^{\frac{1}{x}} = \frac{1}{e} \quad , \quad \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$\forall a \in \mathbb{R}_+ : \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a \quad , \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \quad , \quad \lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$$

$$\forall p \in \mathbb{R}_+ : \lim_{x \rightarrow +\infty} \frac{\ln x}{x^p} = 0 \quad , \quad \lim_{x \rightarrow 0^+} x^p \ln x = 0 \quad , \quad \lim_{x \rightarrow +\infty} (x - \alpha \ln x) = +\infty$$

$$\forall M \in \mathbb{R} : \lim_{x \rightarrow +\infty} \frac{e^x}{x^M} = +\infty \quad , \quad \lim_{x \rightarrow +\infty} x^M e^{-x} = 0$$

$$\gamma_{em} := \lim_{n \rightarrow +\infty} \left(\sum_{j=1}^n \frac{1}{j} - \ln n \right) \approx 0.57721\ 56649\ 01532 \quad \text{costante di Euler-Mascheroni}$$

W30:b. derivate

W30:b.01 Sia $f(x)$ funzione definita in un intervallo reale I a valori reali; si dice **funzione derivabile nell'intervallo I** sse $\forall x \in I : D_x(f(x)) := \frac{df}{dx}(x) := f'(x) := \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} =_{i\exists} \lim_{\Delta x \rightarrow 0} \frac{\Delta}{\Delta}$

$$\forall \alpha \in \mathbb{R} : \frac{d}{dx}(x^\alpha) = \alpha x^{\alpha-1} . \quad \text{in partic. } \forall c \in \mathbb{R} : \frac{d}{dx}c = 0 \quad , \quad \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}} \quad , \quad \frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$$

$$\forall n = 0, 1, 2, \dots : \frac{d}{dx}\sqrt[n]{x} = \frac{1}{n}\sqrt[n]{x^{n-1}} \quad , \quad \forall \beta \in \mathbb{R}_+ : \frac{d}{dx}\frac{1}{x^\beta} = -\frac{\beta x^{\beta-1}}{x^{2\beta}} = -\beta x^{-\beta-1}$$

$$\forall \beta \in \mathbb{R}_+ \setminus \{1\} : \frac{d}{dx}(\beta^x) = \beta^x \ln \beta . \quad \text{in partic. } \frac{d}{dx}(e^x) = e^x \quad , \quad \frac{d}{dx}(e^{-x}) = -e^{-x} \quad , \quad \frac{d}{dx}(e^{\beta x}) = \beta e^{\beta x}$$

$$\frac{d}{dx}(\log_\beta x) = \frac{1}{x} \log_\beta e = \frac{1}{\ln \beta x} \quad , \quad \frac{d}{dx}(\ln x) = D_x(\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x \quad , \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}\sec x = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{\sin x}{\cos^2 x} = \frac{\tan x}{\cos x} \quad , \quad \frac{d}{dx}\csc x = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = -\frac{\cos x}{\sin^2 x} = -\frac{\cot x}{\sin x}$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -1 - \cot^2 x = -\frac{1}{\sin^2 x}$$

$$y = \arcsin x : \frac{d}{dx}\arcsin x = \frac{1}{D_y \sin y} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - x^2}}$$

$$y = \arccos x : \frac{d}{dx}\arccos x = \frac{1}{D_y \cos y} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1 - \cos^2 x}} = -\frac{1}{\sqrt{1 - x^2}}$$

$$y = \arctan x : \frac{d}{dx}\arctan x = \frac{1}{D_y \tan y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

$$y = \operatorname{arccot} x : \frac{d}{dx}\operatorname{arccot} x = \frac{1}{D_y \cot y} = \frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \quad , \quad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}\tanh x = \frac{d}{dx}\left[\frac{\sinh x}{\cosh x}\right] = \frac{\cosh^2 x + \sinh^2 x}{\cosh^2 x} = 1 - \tanh^2 x = \frac{1}{\cosh^2 x}$$

$$\frac{d}{dx}\coth x = \frac{d}{dx}\left[\frac{\cosh x}{\sinh x}\right] = 1 - \coth^2 x = -\frac{1}{\sinh^2 x}$$

$$y = \operatorname{arsinh} x : \frac{d}{dx}\operatorname{arsinh} x = \frac{1}{D_y \sinh y} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1 + x^2}} .$$

$$y = \operatorname{arcosh} x : \frac{d}{dx}\operatorname{arcosh} x = \frac{1}{D_y \cosh y} = \frac{1}{\sinh y} = \frac{1}{\sqrt{1 - \cosh^2 x}} = -\frac{1}{\sqrt{x^2 - 1}}$$

$$y = \operatorname{artanh} x : \frac{d}{dx}\operatorname{artanh} x = \frac{1}{D_y \tanh y} = \frac{1}{1 + \tanh^2 y} = \frac{1}{1 - x^2}$$

$$y = \operatorname{arcoth} x : \frac{d}{dx}\operatorname{arcoth} x = \frac{1}{D_y \coth y} = \frac{1}{1 - x^2}$$

W30:b.02 regole di derivazione

$f(x)$ e $g(x)$ derivabili in x $D_x(\alpha f(x) + \beta g(x)) = \alpha D_x f(x) + \beta D_x g'(x)$

$$\begin{aligned}D_x(f(x) \cdot g(x)) &= D_x f(x) \cdot g(x) + f(x) \cdot D_x g(x) \quad , \quad D_x \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\D_x f(g(x)) &= D_{g(x)}(f(x)) \cdot D_x g(x) \quad , \quad D_x f(g(h(x))) = D_g(f(h(x))) \cdot D_h(g(h(x))) \cdot D_x h(x) \\D_x e^{f(x)} &= e^{f(x)} \cdot D_x f(x) \quad , \quad D_x \ln |f(x)| = \frac{D_x f(x)}{f(x)} \\D_x [f(x)]^{g(x)} &= [f(x)]^{g(x)} \left[g'(x) \cdot \ln f(x) + \frac{g(x) f'(x)}{f(x)} \right]\end{aligned}$$

W30:d. serie numeriche

W30:d.01

$$\begin{aligned}
 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} + \dots &= e \quad , \quad 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \pm \dots = \frac{1}{e} \\
 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + (-1)^{n-1} \frac{1}{n} + \dots &= \ln 2 \quad , \quad 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots = 2 \\
 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots + \frac{(-1)^n}{2^n} + \dots &= \frac{2}{3} \quad , \quad 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots - \frac{1}{4n-1} + \frac{1}{4n+1} - \dots = \frac{\pi}{4} \\
 \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} + \dots &= 1 \quad , \quad 1 + \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} + \dots = \frac{1}{2} \\
 \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{(n-1)(n+1)} + \dots &= \frac{3}{4} \\
 \frac{1}{3 \cdot 5} + \frac{1}{7 \cdot 9} + \frac{1}{11 \cdot 13} + \dots + \frac{1}{(4n-1)(4n+1)} + \dots &= \frac{1}{2} - \frac{\pi}{8} \\
 \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots + \frac{1}{n(n+1)(n+2)} + \dots &= \frac{1}{4} \\
 \frac{1}{1 \cdot 2 \dots h} + \frac{1}{2 \cdot 3 \dots (h+1)} + \frac{1}{3 \cdot 4 \dots (h+2)} + \dots + \frac{1}{n(n+1) \dots (n+h-1)} + \dots &= \frac{1}{(h-1) \cdot (h-1)!} \\
 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots + \frac{1}{n^2} + \dots &= \frac{\pi^2}{6} \quad , \quad 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots + (-1)^{n-1} \frac{1}{n^2} + \dots = \frac{\pi^2}{12} \\
 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots + \frac{1}{n^4} + \dots &= \frac{\pi^4}{90} \quad , \quad 1 - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \dots + (-1)^{n+1} \frac{1}{n^4} + \dots = \frac{7\pi^4}{720} \\
 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots + \frac{1}{n^4} + \dots &= \frac{\pi^4}{90} \quad , \quad 1 - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \dots + (-1)^{n+1} \frac{1}{n^4} + \dots = \frac{7\pi^4}{720} \\
 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{(2n+1)^2} + \dots &= \frac{\pi^2}{8} \quad , \quad 1 + \frac{1}{3^4} + \frac{1}{5^4} + \dots + \frac{1}{(2n+1)^4} + \dots = \frac{\pi^4}{96}
 \end{aligned}$$

W30:d.02 serie per numeri di Bernoulli e di Eulero

$$\begin{aligned}
 1 + \frac{1}{2^{2k}} + \frac{1}{3^{2k}} + \frac{1}{4^{2k}} + \dots + \frac{1}{n^{2k}} + \dots &= \frac{\pi^{2k} 2^{2k-1}}{(2k)!} \text{Brn}_k \\
 1 - \frac{1}{2^{2k}} + \frac{1}{3^{2k}} - \frac{1}{4^{2k}} + \dots + (-1)^{n-1} \frac{1}{n^{2k}} + \dots &= \frac{\pi^{2k} (2^{2k-1} - 1)}{(2k)!} \text{Brn}_k \\
 1 + \frac{1}{3^{2k}} + \frac{1}{5^{2k}} + \frac{1}{7^{2k}} + \dots + (-1)^{n-1} \frac{1}{(2n-1)^{2k}} + \dots &= \frac{\pi^{2k} (2^{2k-1} - 1)}{2(2k)!} \text{Brn}_k \\
 1 - \frac{1}{3^{2k+1}} + \frac{1}{5^{2k+1}} - \frac{1}{7^{2k+1}} + \dots + (-1)^{n-1} \frac{1}{(2n-1)^{2k+1}} + \dots &= \frac{\pi^{2k} (2^{2k-1} - 1)}{2(2k)!} \text{Eul}_k
 \end{aligned}$$

W30:e. successioni e serie di funzioni

W30.f. sviluppi in serie di potenze

In questa sezione assumiamo che sia $\alpha \in \mathbb{R}$ e $a, b \in \mathbb{R}_{nz}$

W30.f.01 sviluppi in serie di potenze di espressioni algebriche

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6} x^3 \cdots + \binom{\alpha}{n} x^n + \cdots \quad \text{per } -1 < x < 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \quad \text{per } -1 < x < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots \quad \text{per } -1 < x < 1$$

$$\frac{1}{a-bx} = \frac{1}{a} \left(1 + \frac{bx}{a} + \left(\frac{bx}{a}\right)^2 + \cdots + \left(\frac{bx}{a}\right)^n + \cdots \right) \quad \text{per } |x| < \frac{|a|}{|b|}$$

$$= -\frac{1}{bx} \left(1 + \frac{a}{bx} + \left(\frac{a}{bx}\right)^2 + \cdots + \left(\frac{a}{bx}\right)^n + \cdots \right) \quad \text{per } |x| > \frac{|a|}{|b|}$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots + (n+1)x^n + \cdots \quad \text{per } -1 < x < 1$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \cdots + \binom{1/2}{n} x^n \quad \text{per } -1 < x < 1$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \cdots + \binom{-1/2}{n} x^n + \cdots \quad \text{per } -1 < x \leq 1$$

W30.f.02 sviluppi in serie di potenze per esponenziali

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \quad \text{per } -\infty < x < +\infty$$

$$a^x = 1 + x \ln a + \frac{(x \ln a)^2}{2} + \frac{(x \ln a)^3}{3!} + \cdots + \frac{(x \ln a)^n}{n!} + \cdots \quad \text{per } -\infty < x < +\infty$$

$$\frac{1}{e^x - 1} = \frac{1}{x} - \frac{1}{2} + \frac{x}{12} - \frac{x^3}{30 \cdot 4!} + \cdots + \frac{\text{Brn}_{2n} x^{2n-1} n}{(2n)!} + \cdots \quad \text{per } -2\pi < x < 0 \text{ e } 0 < x < 2\pi$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots \quad \text{per } -\infty < x < +\infty$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots \quad \text{per } -\infty < x < +\infty$$

$$\tanh x = x - \frac{x^3}{3!} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots + \frac{2^{2n}(2^{2n}-1)}{(2n)!} \text{Brn}_{2n} x^{2n-1} + \cdots \quad \text{per } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} + \cdots + \frac{2^{2n}}{(2n)!} \text{Brn}_{2n} x^{2n-1} + \cdots \quad \text{per } \pi < x < 0 \text{ e } 0 < x < \pi$$

$$\frac{1}{\sinh x} = \frac{1}{x} - \frac{x}{6} + \frac{7x^3}{360} + \dots + \frac{2^{2n}-2}{(2n)!} \text{Brn}_{2n} x^{2n-1} + \dots \quad \text{per } \pi < x < 0 \text{ e } 0 < x < \pi$$

$$\text{csch } x = \frac{1}{\cosh x} = 1 - \frac{x^2}{2} + \frac{5x^4}{24} + \dots + \frac{\text{Eul}_{2n}}{(2n)!} x^{2n} + \dots \quad \text{per } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots \quad \text{per } -1 < x \leq 1$$

$$\ln(a+x) = \ln a + \frac{x}{a} - \frac{1}{2} \left(\frac{x}{a}\right)^2 + \frac{1}{3} \left(\frac{x}{a}\right)^3 - \dots + (-1)^{n-1} \frac{1}{n} \left(\frac{x}{a}\right)^n + \dots \quad \text{per } -a < x \leq a$$

$$\ln(1+x) = \frac{x}{1+x} + \frac{1}{2} \left(\frac{x}{1+x}\right)^2 + \dots + \frac{1}{n} \left(\frac{x}{1+x}\right)^n + \dots \quad \text{per } -\frac{1}{2} < x$$

$$\text{arsinh } x = x - \frac{x^3}{6} + \frac{3x^5}{40} - \dots + (-1)^n \frac{(2n-1)!!}{(2n)!!(2n+1)} x^{2n+1} + \dots \quad \text{per } -1 < x < 1$$

$$\text{arcosh } x = \ln|2x| - \frac{1}{4x^2} - \frac{3}{32x^4} - \dots - \frac{(2n-1)!!}{(2n)!!2nx^{2n}} - \dots \quad \text{per } |x| > 1$$

$$\text{artanh } x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2n+1}}{2n+1} + \dots \quad \text{per } -1 < x < 1$$

$$\text{arcoth } x = \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \dots + \frac{1}{(2n+1)5x^{2n+1}} \quad \text{per } |x| > 1$$

W30:f.03 sviluppi in serie di potenze per espressioni trigonometriche

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots \quad \text{per } -\infty < x < +\infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots \quad \text{per } -\infty < x < +\infty$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots + (-1)^{n-1} \frac{2^{2n}(2^{2n}-1)}{(2n)!} \text{Brn}_{2n} x^{2n-1} + \dots \quad \text{per } \frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\cot x = \frac{1}{x} - \frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} - \dots + (-1)^n \frac{2^{2n}}{(2n)!} \text{Brn}_{2n} x^{2n-1} + \dots \quad \text{per } \pi < x < 0 \text{ e } 0 < x < \pi$$

$$\sec x = \frac{1}{\cos x} = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{6!} + \dots + (-1)^n \frac{\text{Eul}_{2n}}{(2n)!} x^{2n} + \dots \quad \text{per } \frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{37!} + \dots + (-1)^{n-1} \frac{2^{2n}-2}{(2n)!} \text{Brn}_{2n} x^{2n-1} + \dots \quad \text{per } \pi < x < \pi, x \neq 0$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} - \frac{(2n1)!!}{(2n)!!(2n+1)} x^{2n+1} + \dots \quad \text{per } -1 < x < 1$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots \quad \text{per } -1 < x < 1$$

$$\operatorname{arccos} x = \frac{\pi}{2} - \arcsin x \quad \text{per } -1 < x < 1$$

$$\operatorname{arccot} x = \frac{\pi}{2} - \arctan x \quad \text{per } -1 < x \leq 1$$

Alberto Marini

W30:g. prodotti infiniti

Testi dell'esposizione in <http://www.mi.imati.cnr.it/alberto/> e in <http://arm.mi.imati.cnr.it/Matexp/>