1

Capitolo P79
meccanica quantistica: formule

Contenuti delle sezioni

11 pagine

P790.01 Gran parte delle definizioni e dei risultati della meccanica quantistica sono esprimibili mediante formule che spesso risultano alquanto elaborate.
Fine di questo capitolo è una presentazione di definizioni e risultati esprimibili come formule che possa costituire un conveniente riferimento compatto per lo studio della disciplina.

Le formule che seguono si possono distinguere in due categorie.
Quelle della prima categoria riguardano sistemi quantici specifici e strutture generali per la disciplina.
Quelle della seconda classe, inserite per rendere il capitolo un repertorio più autonomo e completo, toccano varie nozioni matematiche (dell'analisi, della geometria e della fisica matematica) che possono essere viste come preliminari delle formule più strettamente quantistiche.

P79 a. valori di aspettazione per 1 dimensione

P79a.01 Funzione d'onda $\Psi(x,t)$ grandezze $f(x,t)$ e $F(x,p,t)$

$$\big\langle f(x,t)\big\rangle \;=\; \int_{-\infty}^{\infty}\Psi^*(x,t)\,f(x,t)\cdot\Psi(x,t) \qquad \big\langle F(x,p,t)\big\rangle \;=\; \int_{-\infty}^{\infty}\Psi^*(x,t)\,F_{op}\left(x,-\mathrm{i}\hbar\frac{\partial}{\partial x},t\right)\cdot\Psi(x,t)$$

$$\langle x\rangle \;=\; \int_{-\infty}^{\infty}\Psi^*(x,t)\,x\cdot\Psi(x,t) \qquad \langle p\rangle \;=\; -\mathrm{i}\hbar\int_{-\infty}^{\infty}\Psi^*(x,t)\,\frac{\partial}{\partial x}\Psi(x,t)$$

$$\langle E\rangle \;=\; \mathrm{i}\hbar\int_{-\infty}^{\infty}\Psi^*(x,t)\,\frac{\partial}{\partial t}\Psi(x,t) \qquad \Delta x \;=\; \sqrt{\langle x^2\rangle-\langle x\rangle^2} \qquad \Delta p \;=\; \sqrt{\langle p^2\rangle-\langle p\rangle^2}$$

potenziale pari $\implies \langle x\rangle \;=\; \langle p\rangle \;=\; 0\ ,\ \Delta x \;=\; \sqrt{\langle x^2\rangle}\ ,\ \Delta p \;=\; \sqrt{\langle p^2\rangle}$

P79 b. coordinate sferiche

P79b.01

$$\begin{aligned} r &= \sqrt{x^2+y^2+z^2} & 0 \le r < \infty \qquad & x = r\cos\phi\sin\theta \\ \theta &= \arccos z/r & 0 \le \theta < \pi \qquad & y = r\sin\phi\sin\theta \\ \phi &= \arctan\, y/x & 0 \le \phi < 2\,\pi \qquad & z = r\cos\theta \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial\, x} &= \cos\phi\sin\theta\,\frac{\partial}{\partial\, r} + \frac{1}{r}\cos\phi\cos\theta\,\frac{\partial}{\partial\,\theta} - \frac{1}{r}\,\frac{\sin\phi}{\sin\theta}\,\frac{\partial}{\partial\,\phi} \\ \frac{\partial}{\partial\, y} &= \sin\phi\sin\theta\,\frac{\partial}{\partial\, r} + \frac{1}{r}\cos\phi\cos\theta\,\frac{\partial}{\partial\,\theta} - \frac{1}{r}\,\frac{\sin\phi}{\sin\theta}\,\frac{\partial}{\partial\,\phi} \end{aligned}$$

$$\nabla^2 = \frac{\partial^2}{\partial\, x^2} + \frac{\partial^2}{\partial\, x^2} + \frac{\partial^2}{\partial\, x^2} = \frac{1}{r^2}\frac{\partial}{\partial\, r}\left(r^2\frac{\partial}{\partial\, r}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\,\theta}\left(\sin\theta\,\frac{\partial}{\partial\,\theta}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial^2}{\partial\,\phi^2}$$

$$\mathbf{p}\cdot\mathbf{p} = -\hbar^2\nabla^2$$

P79 c. oscillatore armonico

P79c.01 Potenziale $V(x) \;=\; \frac{1}{2}\,K\,x^2$ con $K>0$

$$\nu_c \;=\; \frac{1}{2\,\pi}\sqrt{\frac{K}{m}} \qquad \alpha \;=\; \left(\frac{m\,K}{\hbar^2}\right)^{1/4} \qquad E_n \;=\; \left(n+\frac{1}{2}\right)\mathtt{h}\,\nu_c$$

$n \;=\; 0,1,2,\ldots \qquad \psi_n(x) \;=\; N_n\,\mathsf{Hrmt}_n(\alpha\,x)\,\mathsf{e}^{-\frac{1}{2}\,\alpha^2\,x^2} \qquad \xi \;:=\; \alpha\,x \qquad N_n \;=\; \sqrt{\frac{\alpha}{\pi^{1/2}\,2^n\,n!}}$

$\mathsf{Hrmt}_n(\xi) \;=\; (-)^n\mathsf{e}^{\xi^2}\,\frac{\mathrm{d}^n}{\mathrm{d}\xi^n}\,\mathsf{e}^{-\xi^2}$ polinomio di Hermite di grado n

$\mathsf{Hrmt}_0(\xi) \;=\; 1$, $\mathsf{Hrmt}_1(\xi) \;=\; 2\,\xi$, $\mathsf{Hrmt}_2(\xi) \;=\; 4\,\xi^2-2$, $\mathsf{Hrmt}_{n+1}(\xi) \;=\; 2\,\xi\,\mathsf{Hrmt}_n(\xi)-2\,n\,\mathsf{Hrmt}_{n-1}(\xi)$

$\psi_0 \;=\; N_0\,\mathsf{e}^{-\xi^2/2}$, $\psi_1 \;=\; N_1\,\xi\,\mathsf{e}^{-\xi^2/2}$, $\psi_2 \;=\; N_2\left(1-2\,\xi^2\right)\mathsf{e}^{-\xi^2/2}$,
$\psi_3 \;=\; N_3\left(3\,\xi-2\,\xi^3\right)\mathsf{e}^{-\xi^2/2}$, $\psi_4 \;=\; N_4\left(3-12\,\xi^2+4\,\xi^4\right)\mathsf{e}^{-\xi^2/2}$

P79 d. valori di aspettazione per 3 dimensioni

P79d.01 Funzione d'onda $\psi(r,\theta,\phi)$ grandezza $f(r,\theta,\phi)$

$$\left\langle f(r,\theta,\phi)\right\rangle = \int_0^{+\infty} \mathrm{d}r\; r^2 \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi\;\; \psi^*(r,\theta,\phi)\, f(r,\theta,\phi)\, \psi(r,\theta,\phi)$$

Simmetria sferica, cioè $\psi = \psi(r)$

$$\langle x\rangle = \langle y\rangle = \langle z\rangle = \left\langle p_x\right\rangle = \left\langle p_y\right\rangle = \left\langle p_z\right\rangle = 0$$

$$\left\langle x^2\right\rangle = \left\langle y^2\right\rangle = \left\langle z^2\right\rangle = \frac{1}{3}\left\langle r^2\right\rangle$$

$$\left\langle {p_x}^2\right\rangle = \left\langle {p_y}^2\right\rangle = \left\langle {p_z}^2\right\rangle = \frac{1}{3}\left\langle p^2\right\rangle = -\frac{\hbar^2}{3}\left\langle \nabla^2\right\rangle$$

$$\left\langle f(r)\right\rangle = \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} \mathrm{d}r\; r^2\, \psi^*\, f(r)\, \psi(r) = 4\pi \int_0^{+\infty} \mathrm{d}r\; r^2 f(r)\, |\psi(r)|^2$$

$$\frac{1}{3}\left\langle p^2\right\rangle = -\frac{\hbar^2}{3} \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} \mathrm{d}r\; r^2\, \psi^*\, \frac{1}{r^2}\, \frac{\partial}{\partial r}\left[r^2\, \frac{\partial}{\partial r}\psi(r)\right]$$
$$= 4\pi \int_0^{+\infty} \mathrm{d}r\; r^2\, \psi^*\, \frac{1}{r^2}\, \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\psi(r)\right]$$

P79 e. atomo idrogenoide

P79e.01 $\psi_{n,l,m}(r,\theta,\phi) \; = \; R_{n,l}\,Ysph_{l,m}(\theta,\phi) \; =$

$$= -\sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}}\; \mathsf{e}^{-\rho/2}\rho^l\, \mathsf{Plgdra}_{n+l}^{2l+1}(\rho)\; Ysph_{l,m}(\theta,\phi)$$

$n \; = \; 1,2,3,... \quad l \; = \; 0,1,2,...,n-1 \quad m \; = \; -l,-l+1,...,0,...,l-1,l$

$a_0 \; = \; \dfrac{\hbar^2}{\mathsf{e}\,M_e} \quad \rho \; = \; \dfrac{2Zr}{na_0} \quad , \quad E_n \; = \; -\dfrac{Z^2\mathsf{e}^4\,m_e}{2\hbar^2 n^2} \; = \; -\dfrac{Z^2\,\mathtt{h}\,\mathtt{e}}{n^2} \quad , \quad R \; = \; \dfrac{m_e\,\mathsf{e}^4}{4\,\pi\hbar^3 c} \; = \; \dfrac{2\,\pi^2\,m_e\,\mathsf{e}^4}{\mathtt{h}^3\,\mathtt{c}}$

$Llag_\beta^\alpha(\rho) \; = \; \dfrac{\mathrm{d}^\alpha}{\mathrm{d}\rho^\alpha}\left[\mathsf{e}^\rho \dfrac{\mathrm{d}^\beta}{\mathrm{d}\rho^\beta}\left(\rho^\beta\,\mathsf{e}^{-\rho}\right)\right]$ polinomio di Laguerre

$\beta \; = \; 0,1,2,... \quad \alpha \; = \; 0,1,2,...,n$

$Ysph_{l,m}(\theta,\phi) \; = \; (-)^{\frac{|m|-m}{2}} \sqrt{\dfrac{2l+1}{4\pi}\dfrac{(l-|m|)!}{(l+|m|)!}}\,\mathsf{Plgdra}_l^{|m|}(\cos\theta)\,\mathsf{e}^{im\phi}$ funzione sferica di superficie

$\mathsf{Plgdra}_l^{|m|}(\xi) \; = \; (-)^m\,(1-\xi^2)^{\frac{|m|}{2}}\,\dfrac{\mathrm{d}^{|m|}\mathsf{Plgdr}_l(\xi)}{\mathrm{d}\xi^{|m|}}$ funzioni associate di Legendre

$\mathsf{Plgdr}_l(\xi) \; = \; (-)^l\,\dfrac{1}{2^l l!}\,\dfrac{\mathrm{d}^l}{\mathrm{d}\xi^l}(1-\xi^2)^l$ polinomio di Legendre di grado l

Strato K stato $1s$ $\psi_{100} \; = \; \dfrac{1}{\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}\mathsf{e}^{-\rho/2}$

Strato L stati:

$2s \quad \psi_{200} \; = \; \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}(2-\rho)\,\mathsf{e}^{-\rho/2}$

$2p \quad \psi_{210} \; = \; \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}\rho\,\mathsf{e}^{-\rho/2}\cos\theta$

$2p \quad \psi_{21\pm1} \; = \; \mp\dfrac{1}{8\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2}\rho\,\mathsf{e}^{-\rho/2}\sin\theta\,\mathsf{e}^{\pm\mathsf{i}\phi}$

P79 f. momenti angolari ed effetti magnetici

P79f.01 momento angolare orbitale $\vec{L} := \vec{r} \times \vec{p}$ $L_z = x\,p_y - y\,p_x$, ...

$$L^2{}_{(op)} = -\hbar^2 \nabla^2{}_a = -\hbar^2 \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \qquad L_{z\,(op)} = -\mathrm{i}\hbar\frac{\partial}{\partial\phi}$$

autofunzioni degli atomi idrogenoidi senza spin $\psi_{nlm}(r,\theta,\phi)$

$L^2\,\psi_{nlm} = l(l+1)\,\hbar^2\,\psi_{nlm}$ $\quad l = 0, 1, ..., n-1$

$L_z\,\psi_{nlm} = m\,\hbar\,\psi_{nlm}$ $\quad m = -l, -l+1, ..., 0, ..., l-1, l$

orbitali corrispondenti: $l = 0 \Rightarrow s$ $\;l = 1 \Rightarrow p$ $\;l = 2 \Rightarrow d$ $\;l = 3 \Rightarrow f$

$l = 0 \iff \psi_{n00}(r,\theta,\phi) = \psi_{n00}(r) \iff$ simmetria sferica

$\Longrightarrow \langle x\rangle = \langle y\rangle = \langle z\rangle = \langle\vec{r}\rangle = \langle p_x\rangle = \langle p_y\rangle = \langle p_z\rangle = \langle\vec{p}\rangle = 0$

$$\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle = 1/3\langle r^2\rangle \qquad \langle {p_x}^2\rangle = \langle {p_y}^2\rangle = \langle {p_z}^2\rangle = 1/3\langle p^2\rangle = -\frac{\hbar^2}{3}\langle\nabla^2\rangle$$

$$\langle f(r)\rangle = 4\pi\int_0^{+\infty} \mathrm{d}r\; r^2\psi^*(r)f(r)\psi(r)\;\;,\;\;\langle\nabla^2\rangle = 4\,\pi\int_0^{+\infty}\mathrm{d}r\;\;\psi^*\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\psi(r)\right]$$

momento di dipolo magnetico orbitale $\vec{\mu}_l = -\dfrac{g_l\,\mu_b}{\hbar}\vec{L}$

$\mu_b = \dfrac{e\hbar}{2m_e} = 0.927\cdot 10^{-23}\mathrm{Am}^2$ magnetone di Bohr; $\qquad g_l = 1$ fattore orbitale

energia di interazione dipolo - campo magnetico $-\vec{\mu}_l\cdot\vec{B}$

momento angolare intrinseco o spin $\vec{S}$

autofunzioni di L^2, L_z, S^2, S_z : ψ_{nlmsm_s}

$S^2\,\psi = s(s+1)\,\hbar^2\,\psi$ $\qquad$ per l'elettrone $s = \dfrac{1}{2}$

$S_z\,\psi = m_s\,\hbar\,\psi$ $\qquad$ per l'elettrone $m_s = \pm\dfrac{1}{2}$

momento di dipolo magnetico di spin - campo magnetico $\vec{\mu}_s = -\dfrac{g_s\,\mu_b}{\hbar}\vec{S}$, $g_s = 2$

energia di interazione spin - campo magnetico $\pm g_s\,\mu_b\,\dfrac{B}{2}$

deviazione Δz di Stern-Gerlach di atomo di massa M e velocità v_x per il quale

$\dfrac{1}{2}M\,{v_x}^2 = 2\,K_B\,T$ che attraversa magnete di lunghezza X e gradiente $\dfrac{\partial B_z}{\partial z}$:

$$\Delta z = \pm\frac{\partial B_z}{\partial z}\frac{\mu_b\,X^2}{8\,K_B\,T}$$

momento angolare totale $\vec{J} = \vec{L} + \vec{S}$ autofunzioni ψ_{nlsjm_j}

$J^2\,\psi = j(j+1)\,\hbar^2\,\psi$ $\qquad$ per l'elettrone $j = l - \dfrac{1}{2}, l + \dfrac{1}{2}$ (valori semidispari)

$J_z\,\psi = m_j\hbar\,\psi$ $\qquad m_j = -j, -j+1, ..., j-1, j$

$(\vec{S}\cdot\vec{L})\,\psi = \dfrac{\hbar^2}{2}\big[j(j+1) - l(l+1) - s(s+1)\big]\,\psi$

energia di interazione spin - orbita $l \geq 1$ $\quad E_{SL} = \dfrac{1}{2\,{m_e}^2\,\mathsf{c}^2}\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r}\left(\vec{S}\cdot\vec{L}\right)$

per atomi idrogenoidi: $s = \dfrac{1}{2}$, $V = -\dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0}\dfrac{1}{r}$, $E_{SL} = \dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathsf{c}^2}\dfrac{1}{r^3}\left(\vec{S}\cdot\vec{L}\right)$

$$\langle E_{SL}\rangle_{j,l,s} = \frac{\hbar^2}{4\,{m_e}^2\,\mathsf{c}^2}\left[j(j+1) - l(l+1) - s(s+1)\right]\left\langle\frac{1}{r}\frac{\mathrm{d}\,V(r)}{\mathrm{d}r}\right\rangle$$

$$\big\langle E_{SL}\big\rangle_{j,l} = \frac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathsf{c}^2}\big[j(j+1) - l(l+1) - 3/4\big]\left\langle\frac{1}{r^3}\right\rangle$$

$$= \frac{|E_n|\,\alpha^2\,Z^4}{n\,l\,(l+\frac{1}{2})\,(l+1)}\,\frac{1}{2}\,[j(j+1)-l(l+1)-3/4] \quad \left(|E_n| \;=\; \frac{13.607\,\text{eV}}{n^2}\;,\;\alpha \;=\; \frac{\mathsf{e}^2}{4\epsilon_0\hbar c} \simeq \frac{1}{137}\right)$$

P79 g. integrali notevoli per la quantistica

P79g.01 $\int_0^{+\infty} \mathrm{d}x \ x^n \, \mathrm{e}^{-x} \ = \ n!$

$I_m \ := \ \int_{-\infty}^{\infty} \mathrm{d}x \ x^m \, \mathrm{e}^{-x^2} \ = \ \frac{m-1}{2} \, I_{m-2}$

$I_0 \ = \ \sqrt{\pi} \quad , \quad I_1 \ = \ \sqrt{\pi} \quad , \quad I_2 \ = \ \sqrt{\frac{\pi}{2}} \quad , \quad I_4 \ = \ \sqrt{\frac{\pi}{2}} \quad , \quad I_{2\,n} \ = \ \frac{(2n-1)!!}{2^n} \sqrt{\pi}$

$\int_0^{\pi} \mathrm{d}x \ \cos^2 x \ = \ \int_0^{\pi} \mathrm{d}x \ \sin^2 x \ = \ \frac{\pi}{2} \quad , \quad \int_0^{\pi} \mathrm{d}x \ \cos^2 x \ \sin^2 x \ = \ \frac{\pi}{8}$

$\int_0^{\pi/2} \mathrm{d}x \ \sin^n x \ \cos x \ = \ \int_0^{\pi/2} \mathrm{d}x \ \sin x \ \cos^n x \ = \ \frac{1}{n+1}$

$\int_0^{\pi/2} \mathrm{d}x \ \sin^n x \ \cos^k x \ = \ \frac{n!\,k!}{2(n+k+1)!}$

P79 h. numeri quantici e codifiche convenzionali

P79h.01 $n \;=\; 1, 2, 3, ... \quad l \;=\; 0, 1, ..., n-1$

orbitali corrispondenti: $l = 0 \Rightarrow s \;\;,\;\; l = 1 \Rightarrow p \;\;,\;\; l = 2 \Rightarrow d \;\;,\;\; l = 3 \Rightarrow f$

$m \;=\; -l, -l{+}1, ..., 0, ..., l{-}1, l \qquad m_s \;=\; -s, ..., s \qquad j \;=\; l{-}s, ..., l{+}s \qquad m_j \;=\; -j, -j{+}1, ..., j{-}1, j$

per atomi idrogenoidi, alcalini e simili $\quad s \;=\; \dfrac{1}{2} \;\;,\;\; m_s \;=\; \pm\dfrac{1}{2} \;\;,\;\; j \;=\; l - \dfrac{1}{2}, l + \dfrac{1}{2}$

P79 i. effetti magnetici

P79i.01 energia di interazione spin - campo magnetico $\pm g_s\,\mu_b\,\dfrac{B}{2}$

ove $g_s = 2$, $\mu_b = \dfrac{e\,\hbar}{2\,m_e} \ = \ 0.927\cdot 10^{-23}\mathrm{J/T}(= \ \mathrm{A\,m^2})$ magnetone di Bohr

$\Longrightarrow$ B efficace che porta a spaz. spin-orbita $2\,\mu_b\,B \ = \big\langle \Delta E_{SL}\big\rangle \ = \ \mathtt{h}\,\mathsf{c}\left(\dfrac{1}{l} - \dfrac{1}{l'}\right)$

deviazione Δz di Stern-Gerlach di atomo di massa M e vel. v_x $\left(\dfrac{1}{2}M{v_x}^2 \ = \ 2\,K_B\,T\right)$

che attraversa magnete di lunghezza X e gradiente $\dfrac{\partial B_z}{\partial z}$: $\Delta z \ = \ \pm\dfrac{\frac{\partial B_z}{\partial z}\,\mu_b\,X^2}{8\,K_B\,T}$

energia di interazione spin-orbita $l \geq 1$ $E_{SL} \ = \ \dfrac{1}{2\,{m_e}^2\,\mathsf{c}^2}\,\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{dr}\left(\vec{S}\cdot\vec{L}\right)$

per atomi idrogenoidi $s \ = \ \dfrac{1}{2}$, $V \ = \ -\dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0}\left\langle\dfrac{1}{r}\right\rangle$, $E_{SL} \ = \ \dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathsf{c}^2}\,\dfrac{1}{r^3}\left(\vec{S}\cdot\vec{L}\right)$

$\langle E_{SL}\rangle_{j,l,s} \ = \ \dfrac{\hbar^2}{4\,{m_e}^2\,\mathsf{c}^2}\big[j(j+1) - l(l+1) - s(s+1)\big]\left\langle\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r}\right\rangle$

per atomi idrogenoidi $\langle E_{SL}\rangle_{j,l} \ = \ \dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathsf{c}^2}\,[j(j+1) - l(l+1) - 3/4]\left\langle\dfrac{1}{r^3}\right\rangle \ =$

$$\frac{|E_n|\alpha^2 Z^4}{nl(l+\frac{1}{2})(l+1)}\,\frac{1}{2}\big[j(j+1) - l(l+1) - 3/4\big] \quad \left(|E_n| \ = \ \frac{13.607\,\mathrm{eV}}{n^2}\ ,\ \alpha \ = \ \frac{\mathsf{e}^2}{4\,\epsilon_0\,\hbar\,\mathsf{c}} \simeq \frac{1}{137}\right)$$

spaziatura dei livelli dovuta a interazione spin-orbita $\langle\Delta E_{SL}\rangle \ = \ \langle\Delta E_{SL}\rangle_{j_1} - \langle\Delta E_{SL}\rangle_{j_2}$

risoluzione necessaria per osservare separazione SL

$$\frac{\delta\lambda}{\lambda} \ = \ \frac{\langle\Delta E_{SL}\rangle}{E_{\to n}}\ \left(E_{\to n} \ = \ 13.607\,\mathrm{eV}\left(\frac{1}{1^2} - \frac{1}{n^2}\right)\right)$$

L'esposizione in https://www.mi.imati.cnr.it/alberto/ e https://arm.mi.imati.cnr.it/Matexp/matexp_main.php