The energy production profile of a large number of residential co-generators: a statistical evaluation

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Abstract: Data obtained from an extensive experimentation, originally designed to evaluate the comfort and consumption performances of a micro-cogeneration system in a residential context, provide valuable information on the usage pattern of these systems. Using these data, an on/off stochastic process of non-exponential holding times depending on time-varying covariates is estimated. The fitted model provides a building block for the estimation (with uncertainty bounds) of the daily profile of the energy produced by a large number of micro-cogenerators. To this end, a special Monte Carlo simulation technique, based on dependent samples, is devised to describe the energy production in a typical winter day.

Keywords: micro-CHP; aggregate energy production; lognormal holding times; time-varying covariates

1. Introduction

In recent years, distributed generation (DG) has become increasingly attractive to governments and industries worldwide. The reasons for this attention to distributed generation of electricity were initially determined by environmental considerations and marketplace: DG is well suited to the use of renewable and clean energy sources or energy saving through combined heat and power (CHP) generation; liberalization of the energy markets, especially electricity and gas, creates radical changes in electric utilities throughout the world, facilitating the entry of a greater number of small producers.

More recently, as a result of the technological development that has taken place in the field of small power generation systems, particularly in the microturbines and fuel cells systems, a significant contribution to DG is expected to come also from residential micro-CHP systems, which can also be useful to final users, as pointed out by De Paepe et al. [1]. In fact because of rising energy costs, and thanks to recent technological developments of small cogenerators, they have become cost-effective in many world markets, as discussed by Pehnt et al. [2]. Moreover, the increased sensitivity of final users towards energy efficient systems, has stimulated the study of several solutions for the optimization of these applications, as shown by Galli et al. [3] and Mago et al.[4]. JRC-IET [5], on Chapter 7, documents these developments, naming manufacturers of commercial micro-CHP units and reporting countries where very large numbers of small CHPs are already in place (such as Japan, with numbers in the hundreds of thousands and the Netherlands, with 4600).

This scenario opens up new problems especially with regard to the study of electricity generation in a network characterized by a large presence of micro-CHP systems, which follow a not fully predictable on-off pattern depending on factors including climatic variables, the habits of users and the thermal properties of buildings. Argiento et al. [6] consider the point of view of the distribution network operator, under the assumption that a correct daily profile of the aggregate energy production of micro-CHPs is available. This paper concentrates on the energy profile estimation instead, based on data collected from an experimental micro-CHP plant. It provides a model, built from semi-Markov processes with time-varying covariates, whose parameters can be estimated from observed data. Through Monte Carlo simulation the estimated model can be used to assess the production of energy at any given time of the day, along with probability bounds, by taking also into account serial correlation.

There are previous attempts at this in the literature, where the states of the CHP system evolve as a Markov chain or a Markov process, according to whether time is regarded as discrete or continuous.
These studies include for example Hegazy et al. [7] and Kaniktar and Kosanovich [8], where the authors aim at finding some network reliability indexes with stochastic and distributed power sources, so that they do not characterize the energy production profile of CHPs, but rather try to assess their availability. In both papers the Markov assumption refers to the up or down states of a CHP, and it cannot be transferred to the on and off states without modifications. The reason is that the hazard functions of the holding times of on and off reflect the usage pattern of a CHP and they are unlikely to be constant. This study finds in fact that a lognormal holding time probability distribution fits the data very well. As regards the model parameters, Hegazy et al. [7] select parameters based on some engineering guidelines to simulate a practical system and Kaniktar and Kosanovich [8] use values obtained from previous studies, whereas in this paper also parameter estimation from experimental data is performed. The data include also climatic variables and room temperature settings and are derived from experimentations, discussed by Mariotto et al. [9], carried out by ITC-CNR in a test building, originally designed to evaluate the comfort and consumption performances of a micro-cogeneration system in a residential context.

In Section 2 the specific micro-CHP system used for the experiments is described and in Section 3 the way experiments were run and what type of data were collected is illustrated. In Section 4, a stochastic process model based on lognormally distributed holding times is presented: the model represents the alternating sequence of on and off states of the micro-CHP and it depends on time-varying covariates. The model is then fitted by least squares. In Section 5, a special Monte Carlo simulation technique, based on dependent samples, is devised to describe the energy production in a typical winter day. The simulation, using the model fitted in Section 5 as its building block, creates a sequence of similar days, converging to what can be viewed as a stationary 24-hour micro-CHP on/off trajectory, whose pointwise mean and variance are estimated from the simulation itself; then, these pointwise estimates are used to provide the daily energy production profile of a large number of micro-CHPs by a straightforward application of the Central Limit Theorem for random variables. Section 6 concludes the paper by summarizing its main findings.

2. The micro-CHP system

The micro-cogeneration plant installed at the experimental building of ITC-CNR is designed to provide heat to three floors as a simulation of residential customers during the winter.

The system consists of a first internal combustion engine capable of delivering up to 5.5 kW electric power and 12.5 kW thermal power. The system is supplemented by an auxiliary boiler and a storage tank, and is monitored by an acquisition system capable of collecting data from the interior, the exterior and the heating system.
The cogenerator (MICROCHP), the storage tank and the boiler provide heat to three floors (see Figure 1):

- The cogeneration system combines the generation of electricity and heat. The heat is sent to the storage tank by means of the circulation of a fluid (water) through pump B.
- The storage tank provides heat to the building when required and this takes place through the movement of fluid in the floors through a twin pump (pump A). When the thermal demand of users is satisfied, the excess heat is stored.
- The auxiliary boiler acts to meet the peak of thermal demand of the users, so its operation is adequately regulated by the control unit of the cogenerator. In the present work the thermal contribution due to the heat boiler is not considered.

Every floor is equipped with a thermostat, a zone valve or solenoid valve and a heating system made of hot-water radiators: when the temperature set by the user in the zone thermostat is reached, an electrical contact that controls the solenoid is closed and the supply of heat to the floor in question is interrupted.

In the building some electric fan heaters were also installed to simulate the presence of human bodies inside the rooms.

3. Description of experimental data

The experimental data were collected during the heating season with the goal of maximizing the energy efficiency of the building-and-plant system by optimizing the set-up of the control parameters of the cogenerator.

The functioning of the cogeneration system can be summarized as follows.
Using a thermometer, the temperature outside the building is monitored. For any outside temperature, a target temperature of the fluid in the storage tank has been previously assigned. It is possible to act on the operating parameters of the cogeneration system in order to set both the “rule” (control curve) that defines the temperature at which the fluid is heated, and the “protocol” (storage program) to monitor the temperature of the fluid inside the storage tank.

In summary, the operating parameters that can be changed in experiments are:

- storage program: it determines which of the temperatures monitored by the cogenerator will be taken into account to fix a set-point. The set point is a reference for switching the engine off or on.
- control curve: it defines the set-point temperature of the engine as a function of the outdoor temperature
- night-decrement: introduces the possibility to shift the control curve downward at night, in order to fit the curve to the lower heat demand.

A typical shape of the control curve is drawn in Figure 2. It is a broken line consisting of three components. Two horizontal half lines, for those values of the external temperature out of a given range (from a very low to a very high temperature), and a decreasing segment corresponding to the values of the external temperature within the range.

![Control curve](image)

**Figure 2: An example of a control curve.**

The analysis were carried out on data from nine experiments conducted from 9 January 2008 to 6 April 2008. In all the considered experiments, the storage program is the same (the “comfort” program), while different control curves are used. This choice was made in order to store a great amount of heat in the storage tank during the night, making it available in the morning, when there is the peak of heating demand: this operation mode allows a smaller gas consumption, using the stored heat instead of the boiler.

Finally, every experiment is associated with covariates and response variables. A covariate which could be set by the experimenters is the ambient thermostat temperature profile, which was chosen to reflect a residential building pattern:
- 17°C from 0:00 to 6:00
- 20°C from 6:00 to 22:00
- 17°C from 22:00 to 24:00.

Several other variables were monitored during the experiments every ten minutes, in particular:

- The electricity produced from the micro-CHP, a response variable;
- The state (on/off) of the engine at the monitoring instant, a response variable;
- Some environmental variables including temperature, solar irradiation, wind speed, wind direction, relative humidity, representing covariates.

4. The stochastic model for the analysis

For each of the nine experiments we denote by $t$ the number of minutes elapsed from the beginning of the experiment. Since the data were collected every ten minutes, the observations are indexed by grid points $t_i = 10i$ ($i = 0, 1, ..., m$), with $m$ depending on the duration of the experiment.

Let $A(t)$, $t > 0$ be the process describing the state of the cogenerator at time $t$,

$$A(t) = \begin{cases} 
0 & \text{at time } t \text{ the cogenerator is in the off state} \\
1 & \text{at time } t \text{ the cogenerator is in the on state}
\end{cases}$$

Let $E(t_i)$ be the random variable representing the electrical power generated by the cogenerator in the interval $(t_{i-1}, t_i]$. We have that

$$E(t_i) = \int_{t_{i-1}}^{t_i} A(t) P_{\text{nom,e}} dt = \tau_i P_{\text{nom,e}}$$

where $P_{\text{nom,e}}$ is the nominal electrical power of the cogenerator and $\tau_i$ is the time spent in the on state by the cogenerator in the interval $(t_{i-1}, t_i]$. By equation (1), we can recover the sequence of times ($\tau_i$) from the observation sequence ($E(t_i)$).

In particular we made the hypothesis that, if for some $i=1,\ldots,m$ we have $\tau_i < 10$, then the process $A(t)$ jumps at time $t_{i-1}$. With this hypothesis we were able to build

$$(A(t_1), \tau_1); (A(t_2), \tau_2); \ldots; (A(t_m), \tau_m)$$

i.e., the trajectory of the process $A(t)$.

Then, using the data of the nine experiments, we obtained the sequences of time spent by the process $A(t)$ in the on and off states:

$$(T_{11}, T_{12}, \ldots, T_{1n_1})$$
$$(T_{01}, T_{02}, \ldots, T_{0n_0})$$

where the $T_{ij}$’s are the holding times of the process in the on state and $T_{0i}$’s are the holding times of the off state.
Figure 3: Trajectories of some processes from two of the analysed experiments. Experiment (a) went from 9 January 2008 to 13 January 2008; experiment (b) went from 14 January 2008 to 18 January 2008.

In Figure 3 the trajectories of the processes of interest from two experiments are reported; the three curves drawn have been rescaled (the ordinate axis is the electricity production) so that all trajectories are visible. The brown solid line is the setting of the thermostats in residential units, 20 °C or 17 °C. Observe how the processes \( A(t) \) and \( E(t) \) behave differently depending on the setting of the thermostat: the times spent in the on and off states are shorter when the thermostats are set to 17 °C. Moreover the influence of the thermostat setting on the process \( A(t) \) is anticipated. It can happen, for example, that the process jumps to the on state just before the temperature was raised to 20 °C. Then the process remains in this state until the end of the interval with the thermostat at 20 °C. The permanence in the on state is therefore prolonged. We will take this anticipation effect into account in the next section.

5. Results

We modelled the holding times of the states of the process through a lognormal distribution

\[
T_{ji} \sim \log N(\mu_{ji}, \sigma_{ji}^2) \quad j = 0, 1; \ i = 1, \ldots, n_j; \tag{2}
\]

Relation (2) can be written as

\[
\log(T_{ji}) = \mu_{ji} + \epsilon_{ji}, \quad \text{where} \quad \epsilon_{ji} \sim N(0, \sigma_{ji}^2) \quad \epsilon \quad j = 0, 1; \ i = 1, \ldots, n_j. \tag{3}
\]

With each time \( T_{ji} \), we associate a vector of covariates \( x_{ji} = (x_{1ji}, x_{2ji}, \ldots, x_{9ji}) \) with \( j \) in \{0, 1\} and \( i \) in \{1, \ldots, n_j\}. The covariates we considered are measurements of the variables at the jump times of the process \( A(t) \). Then for example, the vector \( x_{ji} \) associated with time \( T_{ji} \) refers to the moment at which the cogenerator is turned off (if \( j=0 \)) or is turned on (if \( j=1 \)). The covariates we considered are:

- \( x_1 = (x_{1ji}) \) outdoor temperature.
- \( x_2 = (x_{2ji}) \) the set-point temperature fixed by the control curve.
• $x_3 = (x_{3ji})$, the setting of the thermostats (17 or 20 degrees), at the jump time of the process plus a forward shift. In particular, for the time spent in the on state we have considered a forward shift of 30 minutes, while for the time spent at the off state the forward shift is 90 minutes.

• $x_s = (x_{sji})$, a categorical variable indicating the month in which the experiment was conducted.

• $x_5 = (x_{5ji})$, a categorical variable indicating whether or not some heathers were installed in the building to simulate the presence of human bodies. It can take on three values: $x_{5ji} = 0$ if no heaters were installed; $x_{5ji} = 1$ with 5.6 kW heaters at each floor; $x_{5ji} = 2$ with 9.6 kW heaters at the first floor and 12.8 kW heaters at the second and third floor.

• $x_6 = (x_{6ji})$, wind speed.

• $x_7 = (x_{7ji})$, solar irradiation.

• $x_8 = (x_{8ji})$, wind direction.

• $x_9 = (x_{9ji})$, humidity.

In defining a model that relates the holding times of the process $A(t)$ in the states on/off to the covariates $x$, equation (3) suggests that we use a log-linear regression setting by letting $\mu_i = x_i \beta$ where $\beta$ is a vector of real parameters.

We considered the holding time in the on state separately from the holding time in the off state. Then we have two regressions:

$$\log(T_{i0}) = \beta_{00} + \beta_{01} x_{1i0} + \beta_{02} x_{2i0} + \ldots + \beta_{09} x_{9i0} + \epsilon_{0i}, \quad \text{where } \epsilon_{0i} \sim N(0, \sigma_0) \text{ and } i = 1, \ldots, n_0,$$

$$\log(T_{ii}) = \beta_{i0} + \beta_{1i} x_{1ii} + \beta_{2i} x_{2ii} + \ldots + \beta_{9i} x_{9ii} + \epsilon_{ii}, \quad \text{where } \epsilon_{ii} \sim N(0, \sigma_i) \text{ and } i = 1, \ldots, n_i.$$  \hspace{1cm} (4)

The least squares parameter estimates are presented in Table 1. The residual standard errors estimates are $\hat{\sigma}_0 = 0.6988$ and $\hat{\sigma}_1 = 0.7601$; the coefficients of determination are $R^2 = 0.4190$ and $R^2 = 0.4173$ for the on and off state regressions, respectively. These values indicate that the set of covariates we used give only a partial explanation of the response variable variation. However, the goodness-of-fit analysis, to be shown below, indicates that our simple regression models are not biased.

<table>
<thead>
<tr>
<th>Regression for the holding time in the on state</th>
<th>Regression for the holding Time in the off state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>Std. Error</td>
</tr>
<tr>
<td>Intercept</td>
<td>4.63E+00</td>
</tr>
<tr>
<td>Outdoor temperature</td>
<td>-1.13E-01</td>
</tr>
<tr>
<td>Set-point of control curve</td>
<td>-3.88E-02</td>
</tr>
<tr>
<td>Wind speed</td>
<td>-4.03E-02</td>
</tr>
<tr>
<td>Irradiation</td>
<td>-2.70E-04</td>
</tr>
<tr>
<td>Wind direction</td>
<td>1.81E-04</td>
</tr>
<tr>
<td>Humidity</td>
<td>-6.48E-03</td>
</tr>
<tr>
<td>Thermostat setting (+30min)</td>
<td>2.02E-01</td>
</tr>
<tr>
<td>Month-February</td>
<td>3.27E-01</td>
</tr>
<tr>
<td>Month-January</td>
<td>9.94E-01</td>
</tr>
<tr>
<td>Month-March</td>
<td>-1.73E-02</td>
</tr>
<tr>
<td>Heaters-1</td>
<td>4.78E-01</td>
</tr>
<tr>
<td>Heaters-2</td>
<td>1.95E-01</td>
</tr>
</tbody>
</table>

In defining a model that relates the holding times of the process $A(t)$ in the states on/off to the covariates $x$, equation (3) suggests that we use a log-linear regression setting by letting $\mu_i = x_i \beta$ where $\beta$ is a vector of real parameters.
We observe that, for both models, the external temperature and the thermostat setting are among the most significant variables. In particular, the effects of these covariates is different in the two regressions.

The outside temperature is negatively correlated with the holding times in the on state and positively correlated with the holding times in the off state. This confirms the intuitive idea that at high outside temperatures the engine of the cogeneration system is idle for longer periods of time, and vice versa at low temperatures.

As far as the thermostat setting is concerned, the sign of the estimated parameters confirms the idea that if 30 minutes after the transition from the off state to the on state the thermostats are at 20 degrees, then we get a larger (on average) holding time in the on state. Vice versa, we get a larger (on average) holding time in the off state when 90 minutes after the transition in the off state the thermostats are at 17 degrees.

For both regressions, looking at the plots of residuals (Figure 4) we believe that there are no problems of heteroskedasticity, while the assumption of normality of the residuals is supported by the “normal probability plots” shown in Figure 5.

Figure 4: Plot of residuals for the two regressions in (4): there are no evident deterministic patterns or signals of heteroskedasticity.
6. Using the model to estimate aggregate electricity production

By using the previously identified model, the goal is to estimate the profile $E(t)$ of the electricity produced by the cogenerator in a typical day on the basis of predicted values of climate variables and on the knowledge of the different settings of the thermostat. Moreover, by studying the process obtained by the superposition of multiple $E(t)$ processes, we can stochastically characterize the amount of electricity fed into a network by a large number of micro-CHPs.

To be more formal, let $\hat{\beta}$ be the vector of estimated parameters in the previous paragraph and let $x(t)$ be the process that represents the value of the covariates in a given day of the year.

We want to study the production of electricity of the cogenerator on that day. Let $\{T_{i1}, T_{i2}, ..., T_{ik}\}$ and $\{T_{k1}, T_{k2}, ..., T_{k0}\}$ be the holding times in the on and off states of the process $A(t)$ during the day. The joint law of these holding times can be factorized as follows:

$$L(T_{11}, ..., T_{ik}, T_{01}, ..., T_{0k}) = L(T_{11}) \times L(T_{01} | T_{11}) \times \cdots \times L(T_{ik} | T_{1i}, ..., T_{i-1}, T_{01}, ..., T_{0i-1}) \times L(T_{0k} | T_{1k}, ..., T_{k1}, T_{01}, ..., T_{0k-1})$$

(5)

Now suppose for simplicity that at 00:00 the cogenerator is active. We have then

$$T_{11} \sim \log N(\hat{\beta}x(0), \sigma^2)$$

After that, the process will stay in the off state for a time having distribution

$$T_{01} \sim \log N(\hat{\beta}x(T_{11}), \sigma^2)$$

In general, if $k < k_1$ and $k < k_0$, conditionally on the previous holding times,

$$T_{ik} \sim \log N(\hat{\beta}x(\sum_{j=1}^{k-1} T_{0j} + T_{1j}), \sigma^2)$$

$$T_{0k} = \log N(\hat{\beta}x(\sum_{j=1}^{k-1} T_{0j} + T_{1j} + T_{ik}), \sigma^2)$$

(6)
The formulas in (5) identify the joint law of the holding times of the process $A(t)$.

In principle, the knowledge of this law allows (see equation (1)) to evaluate the process $E(t) := E(t, x(t))$. In particular we are interested in estimating the profile of the electricity generation during the 24 hours of a typical day, from 00:00 to 24:00, in which the climate variables included in $x(t)$ take an average value and control variables are also set to typical values. Clearly the production of electricity in a typical day depends partly on the weather of previous days, however this dependence becomes weaker as one gets further into the past. In particular, the weather in the hours before 00:00 of the current day is the most influential: if, for example, the system is in the on state at 23:30 because it is very cold outside, it is likely that it will still be in the same state at 00:00. To reflect this dependence without having to consider all possible climate changes of the previous day, we suppose that the values of the covariates on that day are the same as those of a typical day, a reasonable assumption, given that we can expect a similar weather from consecutive days.

By making the same assumption for all the other previous days, we arrive at considering an infinite and periodic sequence of $x(t)$ values with a 24-hour period, which leads us to study the stationary distribution of the trajectories of electricity production from 00:00 to 24:00, as a reasonable representation of what can be expected in a typical day. This distribution can be described as the stationary distribution of the trajectory of a semi-Markovian process, having two states and a holding time distribution which is a function of time-dependent covariates. Maybe it is possible to develop convergence theorems for such processes, but for the purposes of this study it is easier to simulate a large number of typical days for a large number of cogenerators with the Monte Carlo method, add the ordinates of the trajectories of all cogenerators and represent an average of the daily trajectories with empirical 95% probability intervals.

In the following we describe in some detail our simulation procedure:

a) We select $\beta = \hat{\beta}$ and we assume that the micro-CHP moves to the on state at time 0 (midnight) of the first simulated day.

b) For $i=0,1,..,k$ and $j=0,1$, by means of (6), we draw a sequence of holding times as indicated in the scheme below, for $k$ large enough to cover a large number $G$ of days:

\[
\begin{array}{cccc}
T_{11} & T_{12} & \ldots & T_{1k} \\
\downarrow & \downarrow & \ldots & \downarrow \\
T_{01} & T_{02} & \ldots & T_{0k}
\end{array}
\]

c) We discard a certain number of initial holding times from the sampled sequence (to get rid of the effect of the initial condition) and use the rest of the sequence $\{T_{ij}\}$ to construct the trajectory $A(t)$, $t\geq 0$, representing the long-term behaviour of the cogenerator when the covariate function is the periodic $x(t)$, as specified above. Then we can split the sampled trajectory $A(t)$, $t\geq 0$, into a sequence of $G$ adjacent 24-hour bits, which we denote by $A^{(1)}(t), A^{(2)}(t), \ldots, A^{(G)}(t)$, as $0 \leq t \leq 1440$ (recall that we are measuring time in minutes, so 1440 is one day). For any fixed $t$ this sequence is a dependent sample from a Bernoulli distribution with parameter $p(t; \hat{\beta})$, where $p(t; \hat{\beta}), \ 0 \leq t \leq 1440$, is the stationary probability that the micro-CHP is in the on state at time $t$.

d) We may compute an estimate of $p(t; \hat{\beta})$ for any $0 \leq t \leq 1440$ by letting:

\[
\hat{p}(t) := \frac{1}{G} \sum_{g=1}^{G} A^{(g)}(t)
\]
e) We divide the day into equally spaced, small intervals with endpoints given by the sequence $0=t_0, t_1, \ldots, t_m=24$, such that $t_i-t_{i-1}$ is small enough to make the probability of a transition of $A(t)$ within the interval negligible (in our application $t_{i-1}=10$).

f) We suppose, now, that there are $M$ independent cogenerators and let $E_1(t_i),\ldots,E_M(t_i)$ be the energy produced by each cogenerator in the interval $(t_{i-1}, t_i)$. By equation (1) and by the assumption at point e) it follows that

$$E_j(t_i) = (t_i-t_{i-1})P_{\text{nom}}A_j(t_i)$$

where, $A_j(t_i)$ is the state of the $j$-th cogenerator at time $t_i$, as $j=1,\ldots,M$ and $i=1,\ldots,m$.

g) Let now $C(t_i) = \sum_{j=1}^M E(t_i) = (t_i-t_{i-1})P_{\text{nom}} \sum_{j=1}^M A(t_i)$ be the random variable representing the energy produced by the $M$ cogenerators in the interval $(t_{i-1}, t_i)$. Since $C(t_i)$ is the sum of $M$ independent random variables, from the Central Limit Theorem it follows that its distribution can be approximated by a Gaussian. Moreover, we can easily compute $\mathbb{E}(C(t_i)) = M(t_i-t_{i-1})P_{\text{nom}}\hat{p}(t_i, \hat{\beta})$, and $\text{Var}(C(t_i)) = M(t_i-t_{i-1})^2P_{\text{nom}}^2\hat{p}(t_i, \hat{\beta})(1-\hat{p}(t_i, \hat{\beta}))$.

h) Let $z_\alpha$ be the $(1-\alpha)$-quantile of a standard Gaussian distribution; by plugging the estimate $\hat{p}(t)$ obtained at point d) in the mean and variance of $C(t)$ computed at point g) and using the Central Limit Theorem we obtain that: if

$$l(t_i) = M(t_i-t_{i-1})P_{\text{nom}}\hat{p}(t_i) - z_\alpha M(t_i-t_{i-1})P_{\text{nom}}\sqrt{\hat{p}(t_i)(1-\hat{p}(t_i))}$$

and

$$u(t_i) = M(t_i-t_{i-1})P_{\text{nom}}\hat{p}(t_i) + z_\alpha M(t_i-t_{i-1})P_{\text{nom}}\sqrt{\hat{p}(t_i)(1-\hat{p}(t_i))},$$

then $\mathbb{P}\{C(t_i) \in (l(t_i), u(t_i))\} = 1-\alpha$, for each $t_i$ as $i=1,\ldots,m$. That is, $(l(t_i), u(t_i))$ are (approximately) $(1-\alpha)$ probability bounds for $C(t_i)$.

In Figure 6 the main estimate obtained with our simulation procedure has been plotted. The covariate function we considered reproduces the weather and experimental conditions of 10 January 2008. In the graph the mean of $C(t_i)$ (blue solid line) with the probability bounds $l(t_i)$ and $u(t_i)$ (brown dashed lines) are shown. These probability bounds are sufficiently small to conclude that our estimation contains all the information necessary to assess the production of electricity during the day with good precision.
7. Conclusions

For the correct planning and operation of a power system, it is surely useful to have a precise assessment of the aggregate energy production profile of a large number of distributed sources, of which micro-CHPs are an instance. This is what has been obtained in this paper. Since the energy output of many CHPs is strongly dependent on the season, we have provided a method for the assessment of the daily production profile in any season, illustrating an example for a winter day. The specific numerical results depend on the modelling assumption (which has been validated by the experimental data) that holding times of the on and off states of micro-CHPs are lognormal. Further studies in other contexts or experimentations could give some indication on whether this law is specific to our case or it may have some generality.

8. References


