On Bayesian wavelet estimators: Global and Pointwise convergence

Claudia Angelini
Istituto per le Applicazioni del Calcolo "Mauro Picone" - Sez. Napoli, Consiglio Nazionale delle Ricerche, Napoli, Italy.

Abstract

Various Bayesian wavelet estimators have been proposed recently in literature. Following Bayesian approach, a prior distribution is imposed on wavelet coefficients of the unknown response function and a Bayesian estimator is obtained then by applying a suitable Bayesian rule to the resulting posterior distribution of the coefficients. Numerous simulations studies demonstrate the good performance of Bayesian wavelet estimators. However, their frequentist properties and optimality in the minimax sense, in particular, have not been theoretically studied until recently. In this talk we make a step to fill this gap.

First, to capture the sparseness of wavelet expansion, we consider the prior on wavelet coefficients to be a mixture of a mass point at zero and a Gaussian density :

$$w_{jk} \sim \pi_j N(0, \tau_j^2) + (1 - \pi_j)\delta(0)$$

The hyperparameters τ_j^2 and π_j are assumed to be of the form $\tau_j^2 = c_1 2^{-\alpha j}$ and $\pi_j = \min(1, c_2 2^{-\beta j})$. Then, we consider the posterior mean, posterior median and Bayes Factor Bayesian estimators corresponding to the L_2, L_1 and 0/1 losses respectively and are widely used estimators.

As frequentist measures of goodness we consider both global and pointwise mean squared errors and discuss the difference between the two measures regarding the optimality in the minimax sense. For each measure and each estimator we discuss the choice of the hyper-parameters for achieving the optimal performance within Besov spaces. Finally we discuss the Bayesian paradox.

This talk is a summary of join works with F. Abramovich, U. Amato and D. De Canditiis.