

# Joint parameter estimation and biomass tracking in a stochastic predator-prey system

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## Abstract

A Rao-Blackwellized particle filter for estimating the behavioral parameter of the functional response and tracking the biomass of each population in a stochastic predator-prey system is presented in this paper. We consider a predator-prey model with a Lotka-Volterra functional response and small sets of field data. A first validation of the approach has been carried out using synthetic data.

**Keywords:** Prey-predator system; parameter estimation; population tracking; particle filtering.

## 1 Introduction

Successful establishment of biological control strategies is difficult because the current abundance of pest population and properties of the predator functional response, i.e., the *per capita* rate of predation, should be known, but this information is not always available. Moreover, the decision on time and amount of

predator released has to be taken into the dynamical framework of predator-prey interaction.

In this paper, we propose a method for the joint estimation of the dynamical biomass of each population and the feeding rate during the time evolution of population interactions.

## 2 Method

### 2.1 State-space model

We consider the nonlinear state-space model, obtained by the Euler discretization of a stochastic Lotka-Volterra type of model based on [1],

$$\begin{aligned} x_{k+1} &= x_k + \tau [rx_k(1 - x_k) - q_0x_ky_k] - \sigma x_ky_k\Delta w_{k+1}^{(1)} + \varepsilon x_k\Delta w_{k+1}^{(2)}, \\ y_{k+1} &= y_k + \tau [cq_0x_ky_k - uy_k] + c\sigma x_ky_k\Delta w_{k+1}^{(1)} + \eta y_k\Delta w_{k+1}^{(3)}, \\ o_{k+1}^x &\sim \Gamma(x_{k+1}, d_x^2), \\ o_{k+1}^y &\sim \Gamma(y_{k+1}, d_y^2), \end{aligned} \quad (1)$$

where  $x_{k+1}$  and  $y_{k+1}$  are the biomass of prey and predator, respectively, at time  $k+1$  per habitat unit normalized with respect to the prey carrying capacity per habitat unit (plant),  $o_{k+1}^x$  and  $o_{k+1}^y$  are noisy biomass observations defined as Gamma variables with mean equal to  $x_{k+1}$  and  $y_{k+1}$  and variance equal to  $d_x^2$  and  $d_y^2$ , respectively,  $\tau$  is the time step used in the Euler approximation, and  $k = 0, 1, \dots, S$  denotes the discrete time instants. The parameters  $r$ ,  $c$  and  $u$  are species-specific and have been estimated in [1]. The increments of the Wiener processes,  $\Delta w_{k+1}^{(1)}$ ,  $\Delta w_{k+1}^{(2)}$  and  $\Delta w_{k+1}^{(3)}$  are independent Gaussian variables with zero mean and variance  $\tau$ , and the parameters  $\sigma$ ,  $\varepsilon$ , and  $\eta$  have been estimated in [1].

Assume that the parameter  $q_0$  in the functional response  $q_0x_t y_t$  is unknown and the goal is the joint estimation of this behavioral parameter and the biomass variables,  $x_{k+1}$  and  $y_{k+1}$ .

### 2.2 Rao-Blackwellized particle filter

We apply a practical particle filter (PF) to approximate the sequence of posterior probability distributions of the biomass of each population with unknown parameter  $q_0$  given the observations. The proposed algorithm is an example of a Rao-Blackwellized particle filter (RBPF) [2, 3]. Conditional on the sequences  $\mathbf{x}_{0:k}$  and  $\mathbf{y}_{0:k}$ , the estimation of  $q_0$  is solved numerically using a simple Kalman filtering algorithm [4, 5]. The RBPF handles a set of  $M$  particles in the 2-dimensional space of the prey and predator biomass and a bank of  $M$  Kalman filters running in parallel.

This particle filter method is adapted to small observation datasets, updating importance weights and resampling the particle set only when experimental observations become available.

## 3 Experimental results

### 3.1 Dataset simulation

We consider the acarine predator-prey system studied in [1], the prey mite *Tetranychus urticae* and the predator mite *Phytoseiulus persimilis*. The population dynamics is described by Eq. (1) where all parameters are defined in [1] and the behavioral parameter  $q_0$  is unknown.

In order to generate a synthetic dataset, we set  $q_0 = 1.9$ , a time period  $\tau = 1$  day, and a final time  $S = 69$  days. Then we use the model in Eq. (1) to generate sequences of normalized prey  $x_{1:S}$  and predator  $y_{1:S}$  population biomass values. From these complete sequences, we generate eight noisy observations with variance  $d_x^2 = d_y^2 = 10^{-4}$ .

### 3.2 Validation of the RBPF algorithm

We apply the RBPF algorithm with  $M = 10^5$  particles to jointly estimate the unknown parameter  $q_0$  and track the prey and predator biomass given the available set of eight synthetic observations. All particles are initialized in the same way,  $x_0 = 0.1$  and  $y_0 = 0.01$  are set, and a Gaussian distribution<sup>1</sup> is assumed for the prior density of  $q_0$  with zero mean and variance equal to one.

Figure 1 shows the online evaluation of the posterior mean of  $q_0$  generated by the RBPF method. At the final time  $S = 69$ , the value of the posterior mean converges to 1.946 and the posterior variance is 0.025.

For the same simulation, Figure 1 also displays the true (synthetic) sequences  $x_{0:S}$  and  $y_{0:S}$  together with the online biomass estimates. It can be seen that the estimates are accurate at the times where observations are processed, but there is a drift (the error increases) when data are not available, especially for  $k < 40$ . We also see that for  $k \geq 40$  the estimates of  $q_0$  are more accurate, and this also affects the accuracy of the biomass estimation.

## 4 Conclusions

Within the adaptive management framework in Integrated Pest Management (IPM), the predator-prey model we propose can undergo changes leading to improved predictive and explicative capabilities as more information becomes available. Differently from MCMC methods, the PF method does not present restriction on the dataset. In fact, it can be applied also during the period of data collection without waiting up to the end of at least one cycle of the population like in [1]. Compared to standard particle filters, the proposed method reduces

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<sup>1</sup>The proposed methods demand that the prior of  $q_0$  be Gaussian for formal consistency. However, even with the mean of  $q_0$  at  $k = 0$  equal to zero, the inference algorithm performs well; hence, we have chosen to use this prior to illustrate the robustness of the method.

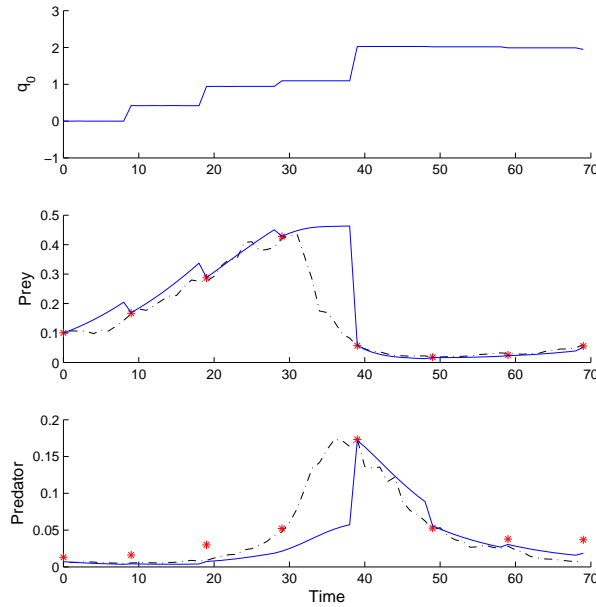


Figure 1: Estimates of the unknown parameter  $q_0$  over time and comparison of the true synthetic biomass sequences (dash-dotted lines) and the online biomass estimates (continuous lines) generated by the RBPF algorithm. The points for which observations are available are displayed with stars.

both the dimension of the state space and the variance of the resulting estimates [2, 3].

The use of the Lotka-Volterra model implies an unsaturated capability of prey biomass intake for the predator. However, the intrinsic limitation in the model is outpaced by the advantages offered by the availability of prompt and progressively improved estimation of the predator functional response.

Finally, the experimental results confirm the goodness of the proposed method. As future work, an application to collected field data will be considered.

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