Reweighting schemes based on particle methods

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Abstract

In SMC methods, the posterior is approximated by a set of properly weighted samples. One problem with such methods is the degeneracy problem: either the weights have huge variability (typically with one or a few samples dominating in weights) or high correlations between the samples. Updating the samples by a few MCMC steps have been suggested as an improvement in this case. The general setup is to first resample the particles in such a way that all particles are given equal weight (resample-move algorithm). Thereafter the MCMC steps are applied in order to make the identical samples diverge. In this work we consider an alternative strategy where the order of a MCMC move and the resampling steps are switched, i.e. MCMC update are performed first. The main advantage such approach is that, followed a MCMC move, the weights can be updated simultaneously making them less variable.

Keywords: Approximate Bayesian inference; Particle filtering; MCMC moves; reweighting schemes; Sequential Monte Carlo methods; State-space models

1 Introduction

Sequential Monte Carlo methods (SMC), and their respective algorithms so-called Particle Filters are efficient classes of Monte Carlo techniques (see [5] for a review) to deal with the intractability in complex models. Typically, we use SMC methods to sample from a high-dimension posterior distribution of interest. Employing SMC methods, the posterior distribution is approximated by a set of properly weighted samples (see [1], [2] and references therein).

Since the seminal paper about SMC methods appeared by [4], the introduction of resample steps arises an alternative to avoid the collapse of particle filter algorithms due to the increasing of dimension. The idea behind adding resampling steps consists of rejuvenating the particles randomly by duplicating the samples with high importance weights and removing samples with low weights. On the other hand, the resample procedure tends to neglect particles with small weights in which may be relevant to describe, for instance, the tail of the target distribution. In order to minimize these undesirable problems after the resample step, MCMC moves within SMC methods (the resample-move approach) were successfully introduced by [3].

In this work, we discuss an alternative strategy for adding MCMC moves in particle filter algorithms before the resample stage. Therefore, our methodology offers a possibility to update the weights and reweight the particles after a MCMC move. Consequently, we can avoid that the distribution of the weights becomes too skewed, and at the same time, we rejuvenate the samples.

2 Discussion of the proposed approach

State-space models provide flexible representations for stochastic dynamical systems in which they encapsulate many real problems (in time or space-time). Let $\{\mathbf{x}_t\}_{t\in\mathbb{N}}$ and $\{\mathbf{y}_t\}_{t\in\mathbb{N}}$ be a discrete-time stochastic process in which the latent process $\mathbf{x}_t$ is indirectly observed through the measurement data $\mathbf{y}_t$. As traditionally used in the literature, the generic state-space models are expressed in terms of an initial distribution, $\mathbf{x}_1 \sim \pi_\theta(\mathbf{x}_1)$, and by some conditional distributions in a hierarchical structure (for $t \geq 1$):

$$\begin{array}{ll} \mathbf{x}_t|\mathbf{x}_{1:t-1} \sim \pi_\theta(\mathbf{x}_t|\mathbf{x}_{t-1}) & \text{Prior Latent Model} \\ \mathbf{y}_t|\mathbf{y}_{1:t-1}, \mathbf{x}_{1:t} \sim \pi_\theta(\mathbf{y}_t|\mathbf{x}_t) & \text{Observation Model} \end{array}$$

All static parameters are represented by $\theta \in \mathbb{R}^p$ and $\mathbf{x}_{1:t} = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_t) \in \mathcal{X}$ and $\mathbf{y}_{1:t} = (\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_t) \in \mathcal{Y}$ denote the first t individuals of the sequence of latent and observed variables. In a Bayesian framework, the knowledge about θ may be described through a prior distribution $\pi(\theta)$.

Let K_t be any transition Markov kernel (see [3] and [5] for features of the MCMC kernel) which leaves $\pi_\theta(\mathbf{x}_{1:t}|\mathbf{y}_{1:t})$ invariant. In a general setting, we can adduce at least four versions for rejuvenation of the particles via move/ new propagate steps on Sequential Monte Carlo framework:

$s.1$ Start with an equally weighted sample, move the particles via K_t and keep the equal particle weights;

$s.2$ Start with a properly weighted sample, move the particles via K_t and keep the same particle weights;

$s.3$ Start with a properly weighted sample, move the particles via K_t and update the particle weights; or

$s.4$ Start with a properly weighted sample, repropagate the particles via a proposal $\tilde{q}_t$ and update the particle weights.

Strategy $s.1$ is well-known the resample-move approach proposed by [3], and $s.2$ is its generalization. Strategies $s.3$ and $s.4$ are cases where, through a diversification step, the weights get updated. Thus, these schemes are promising to make the particle weights less variable and, at the same time, reduce the sample impoverishment. In our framework, we denote these possibilities as *move-reweighting* and *reprograte-reweighting* approaches.

References

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