Approximate Bayesian computation for the elimination of nuisance parameters

Clara Grazian[1]

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[1] University of Rome "La Sapienza", Dipartimento di Scienze Statistiche, Rome, Italy
clara.grazian@uniroma1.it

Abstract

We propose a novel use of the approximate Bayesian methodology. ABC is a way to handle models for which the likelihood function may be considered intractable; this situation is related to the problem of the elimination of nuisance parameters. We propose to use ABC to approximate the likelihood function of the parameter of interest.

Keywords: Approximate Bayesian computation; Integrated likelihood; Markov chain Monte Carlo; Quantile distribution

1 Introduction

Recent developments allow Bayesian analysis also when the likelihood function $L(\theta; y)$ is intractable, that means it is analytically unavailable or computationally prohibitive to evaluate, for instance because of a too high dimension of a latent structure that is part of the model. This methods are known as "approximate Bayesian computation" (ABC) and are characterized by the fact that the approximation of the posterior distribution is obtained without explicitly evaluating the likelihood function. This kind of analysis is popular in genetic and financial settings.

The idea underlying likelihood-free methods is to propose a candidate θ' and to generate a data set from the working model with parameter set to θ'. If the observed and the simulated data are similar "in some way", then the proposed value is considered a good candidate to have generated the data and becomes part of the sample which will form the approximation to the posterior distribution. Conversely, if the observed and the simulated data are too different, the proposed θ' is discarded.

The basic version of the algorithm includes in the posterior sample all the proposal parameters that lead to a distance ρ between a suitable summary statistics $\eta(\cdot)$ computed both on the observed and the simulated data smaller than a tolerance level $\epsilon > 0$. If $\eta(\cdot)$ is a sufficient statistics, then $\lim_{\epsilon \to 0} \pi_\epsilon(\theta;\, \eta(\mathbf{y})) = \pi(\theta;\, \eta(\mathbf{y}))$.

The basic ABC can be inefficient, therefore, ABC algorithms are often linked with other methods, for instance, with MCMC.

2 The elimination of nuisance parameters

In a non-Bayesian setting, the problem of eliminating the nuisance parameters has no general solutions. The idea underlying the available procedures is to accept a partial loss of data information; in general, they require the complete likelihood to be not too complex.

In the presence of a nuisance parameter ϕ, the likelihood function for the parameter θ may be rewritten as

$$L(\theta;\, \mathbf{y}) \propto \frac{\pi(\theta;\, \mathbf{y})}{\pi(\theta)} = \frac{\int_\Phi \pi(\theta,\, \varphi;\, \mathbf{y})\, d\varphi}{\int_\Phi \pi(\varphi)\, \pi(\theta;\, \varphi)\, d\varphi}.$$

Using ABC we may obtain an approximation of $\pi(\theta;\, \mathbf{y})$ constituted by a set of values which may be considered a sample from the posterior distribution. In addition, provided the prior is proper, we are always able to simulate from it. With both a sample from the posterior distribution and a sample from the prior distribution, we can compute an approximation of the likelihood through the ratio of their density estimates.

2.1 Examples

In our work, we discuss some applications of the proposed method. We have always used sufficient summary statistics and the Euclidean distance to compare the observed and the simulated data.

First, we analyze the ABC approximation of the likelihood in situations where other solutions exist: one case where the parameter of interest is a transformation of the parameters of two Poisson distributions and one case from the class of Neyman and Scott; the results are always good approximations of the integrated likelihood function. In general, the tolerance level seems to be a matter of computational power: when ϵ becomes smaller, the approximation is closer to the integrated likelihood, neverthless smaller values are associated to higher computational costs.

Finally, we use the ABC methodology to handle a class of problems with no straightforward solution, that is the pseudo-likelihood for the quantiles of a distribution. In particular, we analyze one kind of quantile distribution: this is a class of distributions defined by their quantile functions that are nonlinear transformations of the quantiles of a standard Normal distribution. This is a class of distribution characterized by a great flexibility of shapes obtained by varying parameters' values and they may model kurtotic or skewed data with the great advantage that they have a small number of parameters, unlike mixture models which are usually adopted to describe this kind of data. It is clear that the density function, and then the likelihood function, are unavailable, therefore any approach but ABC is difficult to implement. In this example, we have compared the basic ABC with ABC-MCMC: these algorithms have different computational costs, neverthless they both lead to good results: we have always obtained approximations concentrated around the true values of the considered quantiles.

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