1

Capitolo W77
promtuario: meccanica quantistica

Contenuti delle sezioni

a. valori di aspettazione in una dimensione p. 2
b. coordinate sferiche p. 3
c. oscillatore armonico p. 4
d. valori di aspettazione in tre dimensioni p. 5
e. atomo idrogenoide p. 6
f. momenti angolari ed effetti magnetici p. 7
g. integrali notevoli per la quantistica p. 9
h. numeri quantici e codifiche convenzionali p. 10
i. effetti magnetici p. 11

11 pagine

W77 0.01 Gran parte delle definizioni e dei risultati della meccanica quantistica sono esprimibili mediante formule che spesso risultano alquanto elaborate.
Fine di questo capitolo è una presentazione di definizioni e risultati della meccanica quantistica che possa costituire un conveniente riferimento compatto per lo studio e le applicazioni della disciplina.

Le formule che seguono si possono distinguere in due categorie.
Quelle della prima categoria riguardano sistemi quantici specifici e strutture generali per la disciplina.
Quelle della seconda classe, inserite per rendere il capitolo un repertorio più autonomo e completo, toccano varie nozioni matematiche (dell'analisi, della geometria e della fisica matematica) che possono essere viste come preliminari delle formule più strettamente quantistiche.

W77 a. valori di aspettazione in una dimensione

W77 a.01 Funzione d'onda $\Psi(x,t)$ grandezze $f(x,t)$ e $F(x,p,t)$

$$\big\langle f(x,t)\big\rangle \;=\; \int_{-\infty}^{\infty} \Psi^*(x,t)\, f(x,t)\cdot\Psi(x,t) \qquad \big\langle F(x,p,t)\big\rangle \;=\; \int_{-\infty}^{\infty} \Psi^*(x,t)\, F_{op}\left(x, -\mathrm{i}\hbar\frac{\partial}{\partial x}, t\right)\cdot\Psi(x,t)$$

$$\langle x\rangle \;=\; \int_{-\infty}^{\infty} \Psi^*(x,t)\, x\cdot\Psi(x,t) \qquad \langle p\rangle \;=\; -\mathrm{i}\hbar\int_{-\infty}^{\infty} \Psi^*(x,t)\,\frac{\partial}{\partial x}\Psi(x,t)$$

$$\langle E\rangle \;=\; \mathrm{i}\hbar\int_{-\infty}^{\infty} \Psi^*(x,t)\,\frac{\partial}{\partial t}\Psi(x,t) \qquad \Delta x \;=\; \sqrt{\langle x^2\rangle - \langle x\rangle^2} \qquad \Delta p \;=\; \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

potenziale pari $\implies \langle x\rangle \;=\; \langle p\rangle \;=\; 0\;,\; \Delta x \;=\; \sqrt{\langle x^2\rangle}\;,\; \Delta p \;=\; \sqrt{\langle p^2\rangle}$

W77 b. coordinate sferiche

W77 b.01

$$\begin{array}{rcll} r &=& \sqrt{x^2+y^2+z^2} & 0 \le r < \infty \\ \theta &=& \arccos z/r & 0 \le \theta < \pi \\ \phi &=& \arctan\, y/x & 0 \le \phi < 2\,\pi \end{array} \qquad \begin{array}{rcl} x &=& r\cos\phi\sin\theta \\ y &=& r\sin\phi\sin\theta \\ z &=& r\cos\theta \end{array}$$

$$\begin{aligned} \frac{\partial}{\partial x} &= \cos\phi\sin\theta\,\frac{\partial}{\partial r} + \frac{1}{r}\cos\phi\cos\theta\,\frac{\partial}{\partial\theta} - \frac{1}{r}\,\frac{\sin\phi}{\sin\theta}\,\frac{\partial}{\partial\phi} \\ \frac{\partial}{\partial y} &= \sin\phi\sin\theta\,\frac{\partial}{\partial r} + \frac{1}{r}\cos\phi\cos\theta\,\frac{\partial}{\partial\theta} - \frac{1}{r}\,\frac{\sin\phi}{\sin\theta}\,\frac{\partial}{\partial\phi} \end{aligned}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\,\frac{\partial^2}{\partial\phi^2}$$

$$\mathbf{p}\cdot\mathbf{p} = -\hbar^2\nabla^2$$

W77 c. oscillatore armonico

W77 c.01 Potenziale $V(x) \;=\; \dfrac{1}{2}\,K\,x^2 \quad$ con $\;K>0$

$$\nu_c \;=\; \frac{1}{2\,\pi}\,\sqrt{\frac{K}{m}} \qquad \alpha \;=\; \left(\frac{m\,K}{\hbar^2}\right)^{1/4} \qquad E_n \;=\; \left(n+\frac{1}{2}\right)\mathtt{h}\,\nu_c$$

$$n \;=\; 0,1,2,... \qquad \psi_n(x) \;=\; N_n\,\mathsf{Hrmt}_n(\alpha\,x)\,\mathsf{e}^{-\frac{1}{2}\,\alpha^2\,x^2} \qquad \xi \;:=\; \alpha\,x \qquad N_n \;=\; \sqrt{\frac{\alpha}{\pi^{1/2}\,2^n\,n!}}$$

$\mathsf{Hrmt}_n(\xi) \;=\; (-)^n \mathsf{e}^{\xi^2}\,\dfrac{\mathrm{d}^n}{\mathrm{d}\xi^n}\,\mathsf{e}^{-\xi^2}$ polinomio di Hermite di grado n

$\mathsf{Hrmt}_0(\xi) \;=\; 1\,,\; \mathsf{Hrmt}_1(\xi) \;=\; 2\,\xi\,,\; \mathsf{Hrmt}_2(\xi) \;=\; 4\,\xi^2-2\,,\; \mathsf{Hrmt}_{n+1}(\xi) \;=\; 2\,\xi\,\mathsf{Hrmt}_n(\xi)-2\,n\,\mathsf{Hrmt}_{n-1}(\xi)$

$\psi_0 \;=\; N_0\,\mathsf{e}^{-\xi^2/2} \quad,\quad \psi_1 \;=\; N_1\,\xi\,\mathsf{e}^{-\xi^2/2} \quad,\quad \psi_2 \;=\; N_2\left(1-2\,\xi^2\right)\mathsf{e}^{-\xi^2/2} \quad,$

$\psi_3 \;=\; N_3\left(3\,\xi-2\,\xi^3\right)\mathsf{e}^{-\xi^2/2} \quad,\quad \psi_4 \;=\; N_4\left(3-12\,\xi^2+4\,\xi^4\right)\mathsf{e}^{-\xi^2/2}$

W77 d. valori di aspettazione in tre dimensioni

W77 d.01 Funzione d'onda $\psi(r,\theta,\phi)$ grandezza $f(r,\theta,\phi)$

$$\big\langle f(r,\theta,\phi)\big\rangle \;=\; \int_0^{+\infty} \mathrm{d}r\; r^2 \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi\;\; \psi^*(r,\theta,\phi)\, f(r,\theta,\phi)\, \psi(r,\theta,\phi)$$

Simmetria sferica, cioè $\psi \;=\; \psi(r)$

$$\langle x\rangle \;=\; \langle y\rangle \;=\; \langle z\rangle \;=\big\langle p_x\big\rangle \;=\big\langle p_y\big\rangle \;=\big\langle p_z\big\rangle \;=\; 0$$

$$\big\langle x^2\big\rangle \;=\big\langle y^2\big\rangle \;=\big\langle z^2\big\rangle \;=\; \frac{1}{3}\,\big\langle r^2\big\rangle$$

$$\big\langle {p_x}^2\big\rangle \;=\big\langle {p_y}^2\big\rangle \;=\big\langle {p_z}^2\big\rangle \;=\; \frac{1}{3}\,\big\langle p^2\big\rangle \;=\; -\frac{\hbar^2}{3}\big\langle\nabla^2\big\rangle$$

$$\big\langle f(r)\big\rangle \;=\; \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} \mathrm{d}r\; r^2\, \psi^*\, f(r)\, \psi(r) \;=\; 4\,\pi \int_0^{+\infty} \mathrm{d}r\; r^2 f(r)\; |\psi(r)|^2$$

$$\begin{aligned}\frac{1}{3}\big\langle p^2\big\rangle \;&=\; -\frac{\hbar^2}{3} \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\phi \int_0^{+\infty} \mathrm{d}r\; r^2\, \psi^*\, \frac{1}{r^2}\, \frac{\partial}{\partial\, r}\left[r^2\, \frac{\partial}{\partial\, r}\psi(r)\right] \\ &=\; 4\,\pi \int_0^{+\infty} \mathrm{d}r\; r^2\, \psi^*\, \frac{1}{r^2}\, \frac{\partial}{\partial\, r}\left[r^2 \frac{\partial}{\partial\, r}\psi(r)\right]\end{aligned}$$

W77 e. atomo idrogenoide

W77 e.01 $\psi_{n,l,m}(r,\theta,\phi) \;=\; R_{n,l}\, Ysph_{l,m}(\theta,\phi) \;=$

$$= -\sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}}\, \mathsf{e}^{-\rho/2}\rho^l\, \mathsf{Plgdra}_{n+l}^{2l+1}(\rho)\, Ysph_{l,m}(\theta,\phi)$$

$n \;=\; 1,2,3,...\quad l \;=\; 0,1,2,...,n-1\quad m \;=\; -l,-l+1,...,0,...,l-1,l$

$a_0 \;=\; \dfrac{\hbar^2}{\mathsf{e}\, M_e}\quad \rho \;=\; \dfrac{2Zr}{na_0}\quad ,\quad E_n \;=\; -\dfrac{Z^2\mathsf{e}^4\, m_e}{2\hbar^2 n^2} \;=\; -\dfrac{Z^2\,\mathtt{h}\,\mathsf{e}}{n^2}\quad ,\quad R \;=\; \dfrac{m_e\,\mathsf{e}^4}{4\,\pi\hbar^3 c} \;=\; \dfrac{2\,\pi^2\, m_e\,\mathsf{e}^4}{\mathtt{h}^3\,\mathtt{c}}$

$Llag_\beta^\alpha(\rho) \;=\; \dfrac{\mathrm{d}^\alpha}{\mathrm{d}\rho^\alpha}\left[\mathsf{e}^\rho \dfrac{\mathrm{d}^\beta}{\mathrm{d}\rho^\beta}\left(\rho^\beta\, \mathsf{e}^{-\rho}\right)\right]$ polinomio di Laguerre

$\beta \;=\; 0,1,2,...\quad \alpha \;=\; 0,1,2,...,n$

$Ysph_{l,m}(\theta,\phi) \;=\; (-)^{\frac{|m|-m}{2}} \sqrt{\dfrac{2l+1}{4\pi}\dfrac{(l-|m|)!}{(l+|m|)!}}\, \mathsf{Plgdra}_l^{|m|}(\cos\theta)\, \mathsf{e}^{im\phi}$ funzione sferica di superficie

$\mathsf{Plgdra}_l^{|m|}(\xi) \;=\; (-)^m\,(1-\xi^2)^{\frac{|m|}{2}}\, \dfrac{\mathrm{d}^{|m|}\mathsf{Plgdr}_l(\xi)}{\mathrm{d}\xi^{|m|}}$ funzioni associate di Legendre

$\mathsf{Plgdr}_l(\xi) \;=\; (-)^l\, \dfrac{1}{2^l l!}\, \dfrac{\mathrm{d}^l}{\mathrm{d}\xi^l}(1-\xi^2)^l$ polinomio di Legendre di grado l

Strato K stato $1s$ $\psi_{100} \;=\; \dfrac{1}{\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \mathsf{e}^{-\rho/2}$

Strato L stati:

$2s \quad \psi_{200} \;=\; \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} (2-\rho)\, \mathsf{e}^{-\rho/2}$

$2p \quad \psi_{210} \;=\; \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \rho\, \mathsf{e}^{-\rho/2}\cos\theta$

$2p \quad \psi_{21\pm1} \;=\; \mp\dfrac{1}{8\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \rho\, \mathsf{e}^{-\rho/2}\sin\theta\, \mathsf{e}^{\pm \mathsf{i}\phi}$

W77 f. momenti angolari ed effetti magnetici

W77 f.01 momento angolare orbitale $\vec{L} := \vec{r} \times \vec{p}$ $L_z = x\,p_y - y\,p_x$, ...

$L^2{}_{(op)} = -\hbar^2\,\nabla^2{}_a = -\hbar^2\left[\dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial}{\partial\theta}\right)+\dfrac{1}{\sin^2\theta}\dfrac{\partial^2}{\partial\phi^2}\right]$ $\qquad L_{z\,(op)} = -\mathrm{i}\hbar\dfrac{\partial}{\partial\phi}$

autofunzioni degli atomi idrogenoidi senza spin $\psi_{nlm}(r,\theta,\phi)$

$L^2\,\psi_{nlm} = l(l+1)\,\hbar^2\,\psi_{nlm}$ $\qquad l = 0,1,...,n-1$

$L_z\,\psi_{nlm} = m\,\hbar\,\psi_{nlm}$ $\qquad m = -l,-l+1,...,0,...,l-1,l$

orbitali corrispondenti: $l = 0 \Rightarrow s$ $l = 1 \Rightarrow p$ $l = 2 \Rightarrow d$ $l = 3 \Rightarrow f$

$l = 0 \iff \psi_{n00}(r,\theta,\phi) = \psi_{n00}(r) \iff$ simmetria sferica

$\implies \langle x\rangle = \langle y\rangle = \langle z\rangle = \langle\vec{r}\rangle = \langle p_x\rangle = \langle p_y\rangle = \langle p_z\rangle = \langle\vec{p}\rangle = 0$

$\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle = 1/3\langle r^2\rangle$ $\qquad \langle {p_x}^2\rangle = \langle {p_y}^2\rangle = \langle {p_z}^2\rangle = 1/3\langle p^2\rangle = -\dfrac{\hbar^2}{3}\langle\nabla^2\rangle$

$\langle f(r)\rangle = 4\pi\displaystyle\int_0^{+\infty}\mathrm{d}r\; r^2\psi^*(r)f(r)\psi(r)$, $\langle\nabla^2\rangle = 4\,\pi\displaystyle\int_0^{+\infty}\mathrm{d}r\;\psi^*\dfrac{\partial}{\partial\,r}\left[r^2\dfrac{\partial}{\partial\,r}\psi(r)\right]$

momento di dipolo magnetico orbitale $\vec{\mu}_l = -\dfrac{g_l\,\mu_b}{\hbar}\vec{L}$

$\mu_b = \dfrac{e\hbar}{2m_e} = 0.927\cdot10^{-23}\mathrm{Am}^2$ magnetone di Bohr; $\qquad g_l = 1$ fattore orbitale

energia di interazione dipolo - campo magnetico $-\vec{\mu}_l\cdot\vec{B}$

momento angolare intrinseco o spin $\vec{S}$

autofunzioni di L^2, L_z, S^2, S_z : ψ_{nlmsm_s}

$S^2\,\psi = s(s+1)\,\hbar^2\,\psi$ $\qquad$ per l'elettrone $s = \dfrac{1}{2}$

$S_z\,\psi = m_s\,\hbar\,\psi$ $\qquad$ per l'elettrone $m_s = \pm\dfrac{1}{2}$

momento di dipolo magnetico di spin - campo magnetico $\vec{\mu}_s = -\dfrac{g_s\,\mu_b}{\hbar}\vec{S}$, $g_s = 2$

energia di interazione spin - campo magnetico $\pm g_s\,\mu_b\,\dfrac{B}{2}$

deviazione Δz di Stern-Gerlach di atomo di massa M e velocità v_x per il quale

$\dfrac{1}{2}\,M\,{v_x}^2 = 2\,K_B\,T$ che attraversa magnete di lunghezza X e gradiente $\dfrac{\partial B_z}{\partial z}$:

$\Delta z = \pm\dfrac{\partial B_z}{\partial z}\dfrac{\mu_b\,X^2}{8\,K_B\,T}$

momento angolare totale $\vec{J} = \vec{L}+\vec{S}$ autofunzioni ψ_{nlsjm_j}

$J^2\,\psi = j(j+1)\,\hbar^2\,\psi$ $\qquad$ per l'elettrone $j = l-\dfrac{1}{2}, l+\dfrac{1}{2}$ (valori semidispari)

$J_z\,\psi = m_j\hbar\,\psi$ $\qquad m_j = -j,-j+1,...,j-1,j$

$(\vec{S}\cdot\vec{L})\,\psi = \dfrac{\hbar^2}{2}\big[j(j+1)-l(l+1)-s(s+1)\big]\,\psi$

energia di interazione spin - orbita $l \geq 1$ $E_{SL} = \dfrac{1}{2\,{m_e}^2\,\mathsf{c}^2}\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r}\left(\vec{S}\cdot\vec{L}\right)$

per atomi idrogenoidi: $s = \dfrac{1}{2}$, $V = -\dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0}\dfrac{1}{r}$, $E_{SL} = \dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathsf{c}^2}\dfrac{1}{r^3}\left(\vec{S}\cdot\vec{L}\right)$

$\langle E_{SL}\rangle_{j,l,s} = \dfrac{\hbar^2}{4\,{m_e}^2\,\mathsf{c}^2}\big[j(j+1)-l(l+1)-s(s+1)\big]\left\langle\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r}\right\rangle$

$\langle E_{SL}\rangle_{j,l} = \dfrac{\mathsf{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathsf{c}^2}\big[j(j+1)-l(l+1)-3/4\big]\left\langle\dfrac{1}{r^3}\right\rangle$

$$= \frac{|E_n|\,\alpha^2\,Z^4}{n\,l\,(l+\frac{1}{2})\,(l+1)}\,\frac{1}{2}\,[j(j+1)-l(l+1)-3/4] \quad \left(|E_n| = \frac{13.607\,\mathrm{eV}}{n^2}\,,\ \alpha = \frac{\mathtt{e}^2}{4\epsilon_0 \hbar c} \simeq \frac{1}{137}\right)$$

W77 g. integrali notevoli per la quantistica

W77 g.01 $\displaystyle\int_0^{+\infty} \mathrm{d}x\ x^n\, \mathsf{e}^{-x} \;=\; n!$

$$I_m \;:=\; \int_{-\infty}^{\infty} \mathrm{d}x\ x^m\, \mathsf{e}^{-x^2} \;=\; \frac{m-1}{2}\, I_{m-2}$$

$$I_0 \;=\; \sqrt{\pi} \quad , \quad I_1 \;=\; \sqrt{\pi} \quad , \quad I_2 \;=\; \sqrt{\frac{\pi}{2}} \quad , \quad I_4 \;=\; \sqrt{\frac{\pi}{2}} \quad , \quad I_{2\,n} \;=\; \frac{(2n-1)!!}{2^n}\sqrt{\pi}$$

$$\int_0^{\pi} \mathrm{d}x\ \cos^2 x \;=\; \int_0^{\pi} \mathrm{d}x\ \sin^2 x \;=\; \frac{\pi}{2} \quad , \quad \int_0^{\pi} \mathrm{d}x\ \cos^2 x\, \sin^2 x \;=\; \frac{\pi}{8}$$

$$\int_0^{\pi/2} \mathrm{d}x\, \sin^n x\, \cos\, x \;=\; \int_0^{\pi/2} \mathrm{d}x\, \sin\, x\, \cos^n x \;=\; \frac{1}{n+1}$$

$$\int_0^{\pi/2} \mathrm{d}x\, \sin^n x\, \cos^k x \;=\; \frac{n!\,k!}{2(n+k+1)!}$$

W77 h. numeri quantici e codifiche convenzionali

W77 h.01 $n = 1,2,3,...$ $l = 0,1,...,n-1$

orbitali corrispondenti: $l=0 \Rightarrow s$, $l=1 \Rightarrow p$, $l=2 \Rightarrow d$, $l=3 \Rightarrow f$

$m = -l,-l{+}1,...,0,...,l{-}1,l$ $m_s = -s,...,s$ $j = l{-}s,...,l{+}s$ $m_j = -j,-j{+}1,...,j{-}1,j$

per atomi idrogenoidi, alcalini e simili $s = \frac{1}{2}$, $m_s = \pm\frac{1}{2}$, $j = l-\frac{1}{2}, l+\frac{1}{2}$

W77 i. effetti magnetici

W77 i.01 energia di interazione spin - campo magnetico $\pm g_s\,\mu_b\,\dfrac{B}{2}$

ove $g_s = 2$, $\mu_b = \dfrac{e\,\hbar}{2\,m_e} = 0.927\cdot 10^{-23}\mathrm{J/T}(=\mathrm{A\,m^2})$ magnetone di Bohr

$\Longrightarrow$ B efficace che porta a spaz. spin-orbita $2\,\mu_b\,B = \langle\Delta E_{SL}\rangle = \mathtt{h\,c}\left(\dfrac{1}{l}-\dfrac{1}{l'}\right)$

deviazione Δz di Stern-Gerlach di atomo di massa M e vel. v_x $\left(\dfrac{1}{2}M{v_x}^2 = 2\,K_B\,T\right)$

che attraversa magnete di lunghezza X e gradiente $\dfrac{\partial B_z}{\partial z}$: $\Delta z = \pm\dfrac{\frac{\partial B_z}{\partial z}\,\mu_b\,X^2}{8\,K_B\,T}$

energia di interazione spin-orbita $l\geq 1$ $E_{SL} = \dfrac{1}{2\,{m_e}^2\,\mathtt{c}^2}\,\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{dr}\left(\vec{S}\cdot\vec{L}\right)$

per atomi idrogenoidi $s=\dfrac{1}{2}$, $V = -\dfrac{\mathtt{e}^2\,Z}{4\,\pi\,\epsilon_0}\left\langle\dfrac{1}{r}\right\rangle$, $E_{SL} = \dfrac{\mathtt{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathtt{c}^2}\,\dfrac{1}{r^3}\left(\vec{S}\cdot\vec{L}\right)$

$\langle E_{SL}\rangle_{j,l,s} = \dfrac{\hbar^2}{4\,{m_e}^2\,\mathtt{c}^2}\big[j(j+1)-l(l+1)-s(s+1)\big]\left\langle\dfrac{1}{r}\dfrac{\mathrm{d}\,V(r)}{\mathrm{d}r}\right\rangle$

per atomi idrogenoidi $\langle E_{SL}\rangle_{j,l} = \dfrac{\mathtt{e}^2\,Z}{4\,\pi\,\epsilon_0\,2\,{m_e}^2\,\mathtt{c}^2}\,[j(j+1)-l(l+1)-3/4]\left\langle\dfrac{1}{r^3}\right\rangle =$

$\dfrac{|E_n|\alpha^2 Z^4}{nl(l+\frac{1}{2})(l+1)}\,\dfrac{1}{2}\big[j(j+1)-l(l+1)-3/4\big]$ $\left(|E_n| = \dfrac{13.607\,\mathrm{eV}}{n^2}\ ,\ \alpha = \dfrac{\mathtt{e}^2}{4\,\epsilon_0\,\hbar\,\mathtt{c}}\simeq\dfrac{1}{137}\right)$

spaziatura dei livelli dovuta a interazione spin-orbita $\langle\Delta E_{SL}\rangle = \langle\Delta E_{SL}\rangle_{j_1} - \langle\Delta E_{SL}\rangle_{j_2}$

risoluzione necessaria per osservare separazione SL

$\dfrac{\delta\lambda}{\lambda} = \dfrac{\langle\Delta E_{SL}\rangle}{E_{\to n}}\left(E_{\to n} = 13.607\,\mathrm{eV}\left(\dfrac{1}{1^2}-\dfrac{1}{n^2}\right)\right)$

Testo fruibile in https://www.mi.imati.cnr.it/alberto/ e https://arm.mi.imati.cnr.it/Matexp/matexp_main.php